IV. *Impact with a Liquid Surface studied by the aid of Instantaneous Photography.* Paper II.

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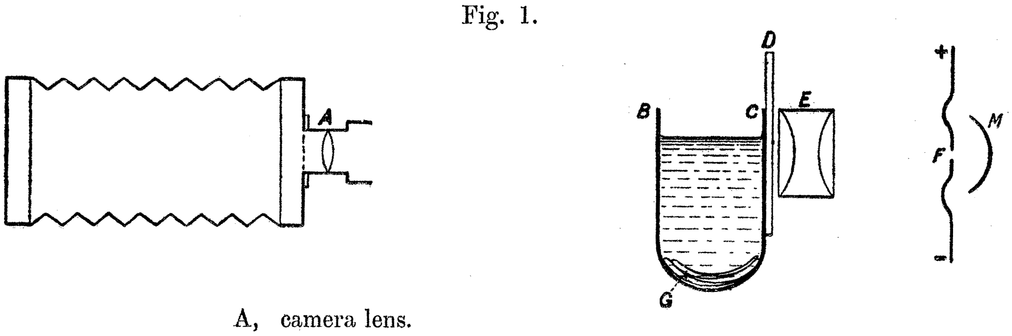
[Plates 2–3.*]

In a previous paper ('Philosophical Transactions,' A, 1897, vol. 189, p. 137) we have drawn attention to the fact that the disturbance set up in a liquid by the impact of a rough sphere falling into it, differs in a very remarkable manner from that which follows the entry of a smooth sphere. In the present paper we describe further experiments, made with the object of ascertaining the reason of this difference, and give the conclusions reached.

It appeared desirable, in the first place, to take instantaneous photographs of the disturbed liquid below the water-line. These were easily obtained by letting the splash take place in an approximately parallel-sided thin glass vessel (an inverted clock-shade) illuminated from behind. The liquid surface when undisturbed was about level with the middle of the camera-lens, which was focussed for the sphere when under water. The general arrangement of the optical apparatus will be sufficiently understood from the accompanying cut (fig. 1). The method of timing the illumination was that already described (*loc. cit.*).

Fig. 1.



A, camera lens.
BC, vessel with liquid.
D, plate of finely roughened glass.
E, condenser taken from an optical lantern.
F, spark-gap at centre of curvature of concave mirror.
M, concave silvered watch-glass.
G, copper gauze to break the fall of the sphere.

* The photographic illustrations accompanying the manuscript of this paper are silver prints mounted on ten sheets, which are referred to as "sheets" in the text. Certain figures from these sheets have been selected for collotype reproduction, and are given on Plates 2 and 3. Others are given in the text. When a figure is referred to which is not reproduced either by photography or by a cut in the text, the No. of the figure is enclosed in [ ].

The first series of these photographs (see Sheet 1, from which a selection is reproduced on Plate 2) is, for the sake of clearness, called Series X., so as to keep the numbering continuous with the previous paper. The photographs show what is going on below the surface of the liquid while the phenomena of Series VIII. of the previous paper are visible above the surface.* The sphere was a rough marble, freshly sand-papered on each occasion.

The photographs 1 and [2] show the sphere gradually entering the surface ; above it is the inverted image of the part that has entered, formed by internal reflection on the liquid surface. Higher up still (in fig. 1) and not quite in the same vertical line (on account of a slight optical displacement due to the front of the vessel not having been set quite perpendicular to the line of sight) is seen the top of the sphere. This, however, is out of focus, for the camera was focussed on the part under water, which is optically brought forward.

In figs. [3, 4] and 5 it will be observed that the liquid already leaves the sphere along a tangent, and from this point onward the sphere is followed by a bag or pocket of air of gradually increasing depth. The wall of this pocket is not quite smooth, and the summit of the sphere, as seen through it, is always somewhat distorted. The sharp angle made with this wall by the oppositely sloping wall of the image tells in each figure the position of the surface, and in several figures we see the lobed lip of the crater that has been already photographed from above in Series VIII. The present photographs show the exact height of this. It will be observed that the depth of the crater or pocket below the surface is far greater than the photographs of Series VIII. gave any reason to suspect, also that the upper part of the sphere is not wetted at all.†

We can find no trace of any reflecting layer of air between the lower part of the sphere and the water, and have no reason to doubt that the lower part is thoroughly wetted up to the place at which the liquid is seen to leave it. As the sphere descends the position of the circular line of contact rises on the sphere and the liquid does not always leave it along a tangent. The ripple-marks conspicuously visible in fig. 9, and in many of the later figures, are indications of the flow of the liquid along the walls of the cylinder of air. The long cylindrical hollow that is thus formed is not a configuration of stable equilibrium, and, if we may leave momentum out of account, its law of spontaneous segmentation will be the same as for a liquid column of the

* This series of photographs, with the exception of the last two, was taken in June, 1896, and was exhibited at the soirée of the Royal Society in June, 1897.

† The present photographs, taken in conjunction with those of Series VIII. of Paper I., bring out a point which had escaped us. For it is now evident that the hollow below the surface in such figures as 8, 9, and 10 of Series VIII. is far too deep and capacious to be filled by the small amount of liquid raised above the general level at the edge of the water, and can, therefore, only be accounted for by a *rise of the general level*, extending to a very considerable distance from the splash. We showed that this phenomenon accompanied the entry of a smooth sphere ; we now see it to be even more marked with a rough one.

same dimensions—in air. Thus the cylinder finally divides into two portions, one of which follows in the wake of the sphere as an attached bubble (figs. 16 and 17), while the other rapidly fills up, apparently, or at any rate in part, by the pouring in of liquid round the rim of the basin. The convergence of this inward flow corresponds to an increasing velocity as the axis is approached, and results in the very rapid upward spirt of the jet that is so well shown in Series VIII. of the previous paper.

Our opinion that the basin does fill up by a motion of this kind is based upon the evidence of the photographs of Series XII., which will be explained shortly. Meanwhile it is convenient to point out that Series XI. (here shown by drawings) gives the under-water phenomena corresponding to Series IX. of the previous paper. In this the sphere (still *rough*) is let fall from a greater height, 50 centims., and the crater thrown up closes and forms a bubble which subsequently opens again and makes

Series XI.

Fig. 1.

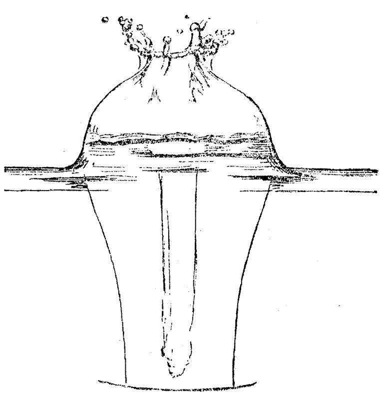


Fig. 2.

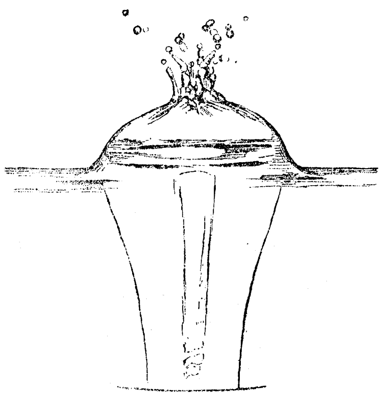


Fig. 3.

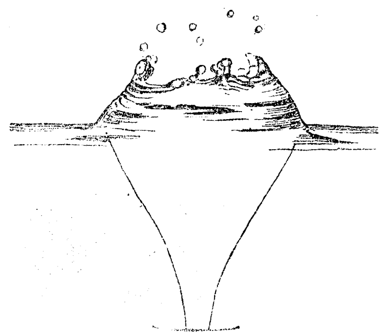


Fig. 4.

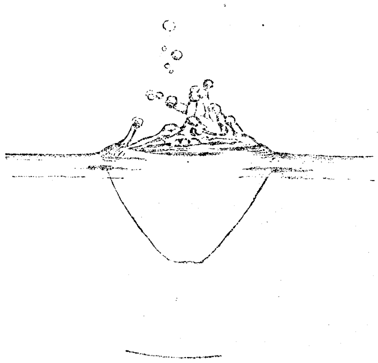
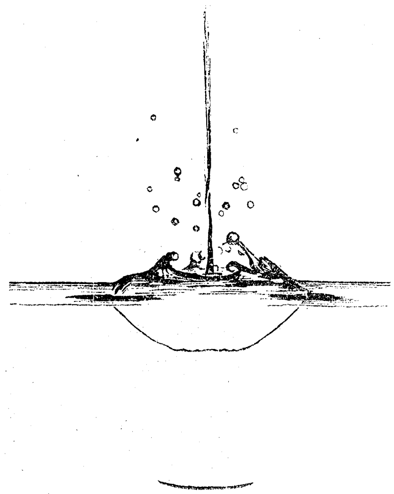


Fig. 5.



way for the emergent jet (*cf.* Series II., figs. 16, 17, and 1A, and Series III., fig. 18, of the earlier paper).

In order to obtain, if possible, some information as to the actual motion of the liquid, a variety of experiments was made with coloured bands and floating bodies, and finally we hit upon the method of letting the sphere fall into dilute sulphuric acid between two electrodes (the bared extremities of two insulated vertical copper wires), from each of which there ascended slowly a vertical stream of very minute bubbles liberated by electrolysis. The velocity of ascent of these bubbles, though it was not always quite slow enough to avoid eddying motion, was so slow (about $1\frac{1}{2}$ centims. per second) that the gravitative displacement of any one bubble during a splash is practically negligible, and the line of bubbles may be therefore taken as a line of marked liquid whose displacement can thus be studied, for the bubbles are so small that it may, we think, be safely assumed that they did not in any way interfere with the motion of the neighbouring liquid. On account of this minuteness the stream of bubbles is not always easy to see in the photographs, and in printing from the negatives long exposure in bright sunshine is necessary.

It will be observed that whereas in fig. 1 of Series XII. (here reproduced) each stream is delivered from the top of the electrode; in figs. 2 and 3 the stream has been swept off the electrode, especially on the left-hand side where the electrode is

Series XII.

Fig. 1.

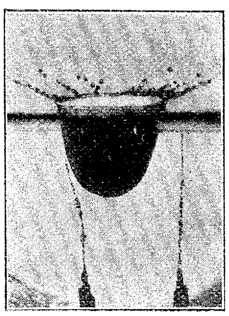


Fig. 2.

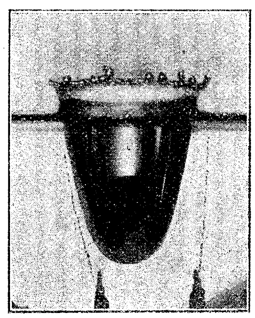


Fig. 3.

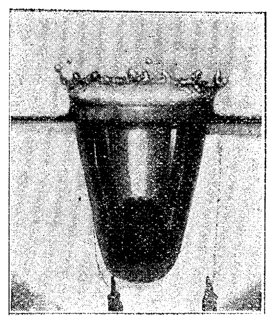


Fig. 5.

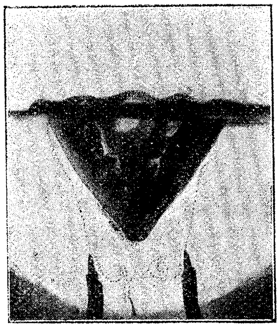
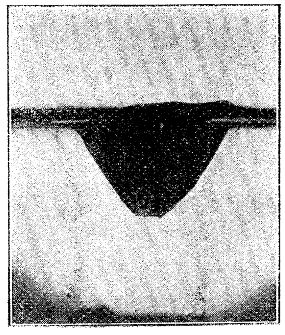


Fig. 6.



nearer the sphere. Fig. 5 shows that the division of the air column is accompanied by the formation of an annular vortex. Fig. 6 [7, 8, and 9] show the filling up of the crater. The vertical lines of bubbles recover their original positions ; but at the top, where the lateral displacement has been greatest, it will be observed, especially in fig. 6, that the stream of bubbles turns over inwards, and it is on this that we rely as proving that in part, at any rate, the hollow fills up by the pouring in of liquid down the sides. There are unmistakable indications of this same curling over in the negative of fig. [8] when the jet has begun ; but the stream of bubbles is so faintly visible that it would disappear in any reproduction.

The next step in the investigation was to examine in more minute detail the splash of a *smooth* sphere. For this purpose we procured a number of very hard highly polished steel bearing balls of three sizes,* viz. :—

(1) $\frac{6}{8}$ inch (= 19·0 millims.) in diameter.
(2) $\frac{5}{8}$,, (= 15·9 ,,) ,,
(3) $\frac{4}{8}$,, (= 12·7 ,,) ,,

Several of each size were subsequently coated with an electrolytic deposit of nickel, which took a still higher polish.

In the year 1897 several hundred splashes were observed with these spheres. Previous observations, as recorded in Series IV., V., and VI. of the earlier paper, had shown that when a well-polished sphere enters with a not very great velocity, the liquid rises over it in a thin sheath, covering it completely before it is entirely below the level of the surface, and that such a sphere consequently takes down no air whatever into the liquid ; also that the motion of the liquid is different, from the earliest moments of contact, from that set up by the entry of a rough sphere. We had, however, noticed that with a much-increased height of fall the smooth sphere took down air and behaved like a rough one.

It appeared possible to us that there was some critical velocity which might mark the change very sharply, and accordingly our first observations were directed to the examination of this point. It was not necessary for this to use any instantaneous illumination ; the magnetically-controlled "catapult" release was still employed for letting the spheres fall, but the observations were conducted in the full light of the laboratory. The results of some hundreds of observations may be summarised as follows :—

With any one sphere always carefully polished in the same way just before letting it fall, a height of fall is soon reached at which the splash ceases to be "completely airless" or smooth, but is followed by the rise of bubbles, at first a shower of very

* We afterwards, as will be seen, employed spheres, 25·7 millims. in diameter, of polished serpentine, and may here state at once that we have not found that the size of the sphere has, within the limits mentioned, any noticeable influence on the course of the splash.

small and inconspicuous bubbles, afterwards when the height is increased, of large and very visible bubbles.

Thus the transition from "smooth" to "rough" is gradual. But the equilibrium of the splash, if we may use the phrase, is very unstable and very much depends on the condition of the surface of the sphere when dropped in.

Thus a polished steel sphere 15·9 millims. in diameter was found to give an airless splash when falling into water from a height of 132·5 centims.; at 137·5 centims. there was much air taken down. This observation at 137·5 centims. was repeated three times, Observer C doing the polishing. Then Observer W polished, and the splash was first *nearly* airless, then *quite* airless. Then, by persevering in the rubbing, the height of fall was gradually raised to 162·5 centims., and a perfectly airless splash was secured; and even at 172·5 centims. the record was "very little air indeed."

Again, a polished marble sphere 2·57 centims. in diameter falling into water from a height of 112 centims. was found to take "much air" when rubbed with clean handkerchief, A, and "none at all, or only very little," when rubbed with clean handkerchief, B. This result was confirmed four times with B, and five with A. These handkerchiefs were subsequently examined under the microscope, but were found to be extremely similar, and the cause of the difference remained for the time beyond conjecture.

On another occasion, of two similar nickel-plated steel spheres, each 19 millims. in diameter, and each treated in exactly the same way, falling 22 centims. into paraffin oil, one would always take down much air and the other little or none, and again microscopic examination showed only a very slight difference in the surfaces.

Influence of the Nature of the Liquid.

The nature of the liquid employed has a great influence in determining whether at a given height the splash shall be "rough" or "smooth."

Thus with paraffin oil the maximum height that could be reached with an airless splash with highly polished nickel-plated spheres, well rubbed on a selvyt cloth, was found to be only 24·7 centims., but with water a fall of 160 centims. could be reached. Whenever water was used as the liquid it was contained in a deep glass bowl, kept brim-full and running over by means of an india-rubber supply pipe from the main, so that the surface was kept perfectly clean. The Devonport water is drawn from a granite country, and is very soft and pure.

We shall revert later to the manner in which the physical constants of the liquid come into play.

Influence of Temperature.

We then found that if the polished sphere was heated in boiling water, quickly rubbed dry, and let fall while it was still hot, a very marked difference was produced.

With the polished sphere hot, the height of fall can be much increased before the splash becomes "rough." Thus, with paraffin oil, the height with a nickelled sphere rose from 22·2 centims. to 29·3 centims., and with water from 157 centims. to 234 centims.

Influence of a Flame held near the Liquid, and traversed by the Sphere in its Fall.

It then occurred to us to let the sphere drop through a flame held near the liquid, and the result was very remarkable. With paraffin oil (and the sphere hot) the airless height now rose from 29·3 centims. to 45·3 centims., and with water and a cold sphere, it rose from 157 centims. to over 258 centims., which was the greatest height that the laboratory would permit. Either the luminous flame of a bat's wing burner or the flame of a Bunsen burner held nearly horizontal produces the effect, provided the flame is held near enough to the surface of the liquid, and it is a very striking experiment to let the polished sphere fall several times from a height which gives a large volume of bubbles rising with much noise to the surface, and then to let it fall through the flame, and to observe the complete change in the phenomenon.

Electrification.

It seemed to us extremely probable that we had here to deal with an electrical phenomenon, for a flame would certainly discharge completely any electrified sphere passing through it, and it appeared reasonable to suppose that the sphere became electrified by friction when falling through the air. Experiments were therefore made to test this supposition. Holding the flame high above the surface should diminish the effect, for the sphere would become again electrified in the remainder of its fall. Experiment abundantly confirmed this view. Thus, with the maximum height of fall available (258 centims.), the flame was still quite effective at 68 centims. above the surface of flowing water, but quite ineffective at 113·5 centims.

Nevertheless, other tests failed to confirm this theory of electrification. If the difference is due to electrification, we argued, then it ought to become very conspicuous when the sphere is deliberately electrified. Accordingly, a long series of experiments was made, in which the sphere soon after its release came into contact with a flexible wire brush connected to a Wimshurst machine, and thus was electrified positively or negatively at pleasure. A height of fall was then chosen for which the splash was either just airless or just not airless when the sphere was unelectrified, so that the influence of electrification in changing the character of the splash either way could be observed.

The results of these observations were curiously discordant,* and though it is

* The probable reason will be mentioned later.

proverbially difficult to prove a negative, the conclusion was finally forced upon us that this electrification, whether positive or negative, had no certain or direct influence. We also tried the effect of holding a charged ebonite rod near the splash, but with negative results.

One test we were able to make which appears to us to be crucial. If the flame acts by diselectrifying the sphere, then the same result should follow from letting the sphere touch in its fall an earth-connected brush near the water, but such contact had no observable effect whatever, and we therefore came to the conclusion that the action of the flame was not an electrical discharging action at all.

In this connection also may be mentioned two other facts: (1) That the flame had no observable effect on a roughened sphere; and (2) that we could not detect any accumulation of electricity when we let the sphere fall time after time through a tube connected to an electroscope, to which tube the sphere could give up its whole charge by touching a wire brush in the interior.

Photographs of the Transition from "Smooth" to "Rough."

Having found that the splash passed by gradual transition from "smooth" to "rough" as the height of fall of a polished sphere was increased, we decided to obtain a photographic record of the process. Series XIII., Sheet 3, of which a selection is here reproduced on Plate 2, shows a perfectly smooth splash produced by a highly polished sphere of serpentine, just over 1 inch (2·57 centims.) in diameter, falling into water from a height of between 14 and 15 centims.

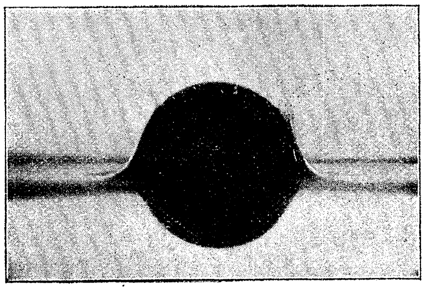
The earlier figures of this series show how extremely thin is the enveloping sheath in its early stages. In fig. 1 it is almost best seen by its reflected image in the undisturbed surface. A nearly horizontal row of minute drops may be seen in figs. [2] and 3. As we shall see later, the place of origin of any one of these is to be found very approximately by drawing a tangent to the sphere through the drop in question in the plane containing the axis, for unless the velocity of the sphere is very materially altered after the moment of separation, the drop will remain on the tangent along which it was projected. The "lug" at either side in figs. [2] and 3 is probably not really the continuous jet it seems to be, but merely the fore-shortened edge of a ring of separate droplets, as is more apparent in figs. [4] and 5. Where the horizontal equator of the sphere is passed, the film will thicken by convergence, and the number of segmentations will diminish accordingly, the drops becoming larger and fewer. It will be noticed that in each of the figs. 5 and 6, a vertical tangent to the sphere marks the limit of the larger drops in the air above.

Fig. 8 shows by means of bubbles that the displacement of the liquid corresponds approximately to stream-line flow in a perfect fluid.

Series XIV., Sheet 4, of which fig. 1 is given on p. 183, shows the effect of increasing the height of fall to 50 centims. It will be observed that the minute jets* projected from the edge of the film are now much higher, while in Series XV. (see fig. 2 here reproduced), in which the height of fall was raised to 75 centims., we see that the

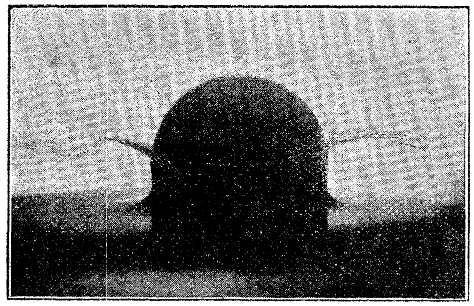
Series XIV.

Fig. 1.



Series XV.

Fig. 2.



film has left the sphere at an earlier stage, and has more nearly the configuration observable in a rough splash. In Series XVI., of which figs. 1, 2, and 3 are reproduced in Plate 3, the height of fall was 100 centims., and the change was still more marked. No air is taken down by this splash.

In order to avoid the necessity of a somewhat inconveniently great height of fall, which would have been imposed by the further use of water as the liquid, we employed Alexandra oil for watching below the surface the beginning of the process by which the sphere takes down air in its wake. Thus Series XVII., Sheet 5, here reproduced in the text, shows the splash of a polished nickelled-steel sphere

Series XVII.

Fig. 1.

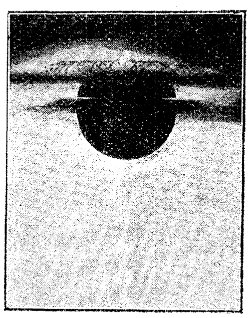
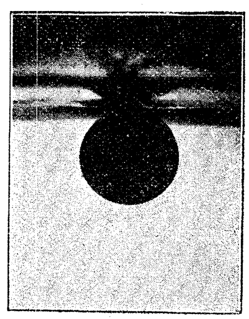


Fig. 2.



* The photographs (unfortunately not the reproductions here given) both of this and of our previous paper illustrate incidentally the great rapidity with which fine jets undergo division into drops, which, however, as Lord RAYLEIGH has explained ('Roy. Soc. Proc.,' vol. 29, 1879, p. 85, on the Capillary Phenomena of Jets), need cause no surprise, since the time of complete segmentation will vary inversely as the 3/2 power of the diameter—if viscosity does not hinder.

Series XVII.

Fig. 3.

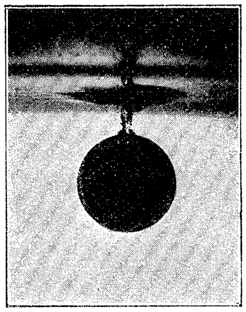
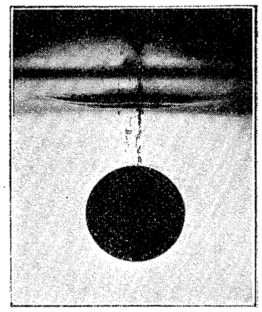


Fig. 4.



19 millims. in diameter falling 30 centims. into Alexandra oil, a height at which ordinary observation in continuous light had shown that a little air was already taken down. In Series XVIII. the height was 40 centims., and in Series XIX. it was 50 centims.

Series XVIII.

Fig. 1.

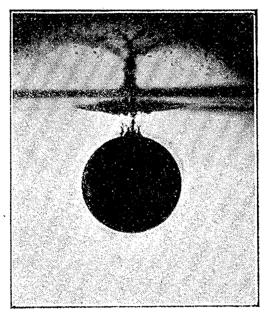


Fig. 2.

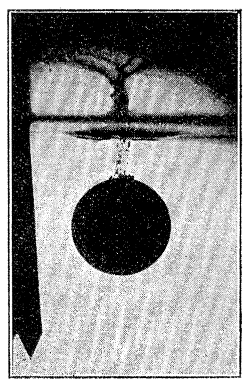


Fig. 3.

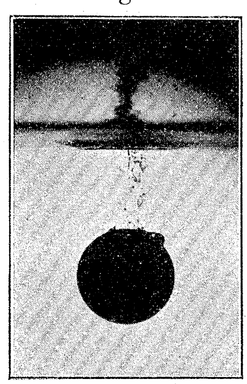
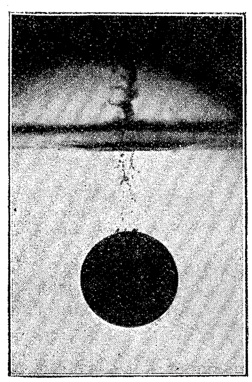


Fig. 4.



Series XIX.

Fig. 1.

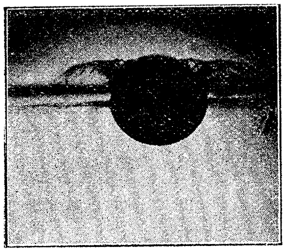


Fig. 2.

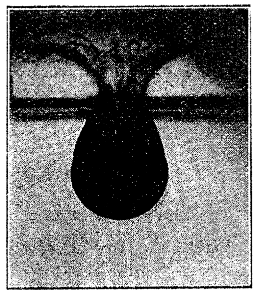


Fig. 3.

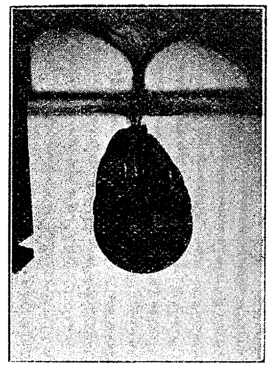


Fig. 4.

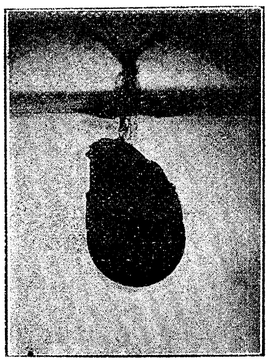


Fig. 5.

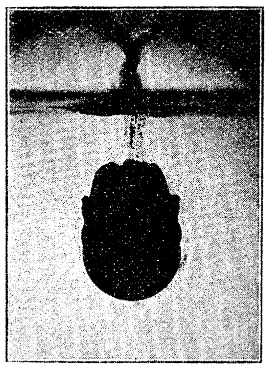
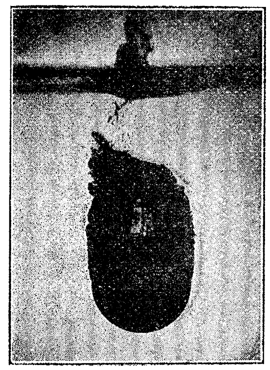


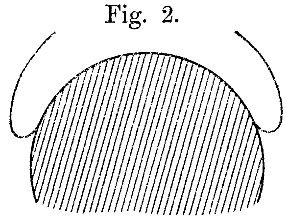
Fig. 6.



In every case there is a quasi-stream-line convergence of flow in the rear of the sphere which was not observable in the rough splash ; this sweeps together the bases of the jets in the crater above the surface, and the entrapping of air seems to depend on the depth to which the sphere has descended when the convergence is completed, and cannot now be attributed, as in the "rough splash," to the spontaneous segmentation of a cylindrical cavity too long to be in stable equilibrium.

It should be mentioned that the bubble is liable to detach itself from the sphere, either wholly or in part, before any considerable depth is reached ; thus, in figs. 2 to 6 of Series XIX., careful examination of the photographs with a lens shows that the liquid is beginning to pass along the surface of the sphere between it and the air in the manner indicated (with exaggeration) in the accompanying cut (fig. 2).

Fig. 2.



The minute but deep corrugations on the surface of the air-bubble in fig. 6 are probably indications of rapid turbulent motion at the interface.

We may here recall to the recollection of the reader that a remarkable feature of the sheath enveloping a smooth sphere, entering with low velocity, was the strongly accentuated radial ribs and flutings (*cf.* Series XXVI., figs. 1 and 3, p. 196), which are specially well seen in the figs. 3, 4, 5, and 9 of Series VI. of Paper I. The present photographs, taken, as they are, with the light behind the object, do not bring these out very well, but traces of them may be detected with a lens in the original photographs, though not in the present reproductions, on the under surface of the liquid to the left of the sphere in figs. 1 and 2 of Series XVII., and in figs. 1 and 2 of Series XVIII., on the right side of the sphere in fig. 1 of Series XIV., and again in fig. 4 of Series XVI.; and it must not be forgotten that they are probably always present. Of this fluting we shall be able presently to give a pretty complete account.

General Explanation of the Phenomena.

The explanation which seems to give the key to the whole phenomenon was suggested (1) partly by the observation of AITKEN that dust does not settle on an object hotter than the air, (2) partly by the observation of QUINCKE that a film of extremely small thickness spreads with great rapidity by molecular action over a polished surface, *e.g.*, of glass or mica when this is touched by a liquid, and (3) partly by our own observation that, at any rate in the neighbourhood of the surface, a flow once set up along any channel is comparatively persistent and determines the motion that is to follow. As illustrations of this we may cite the regular disposition of jets round the rim of the crater thrown up by an entering drop, which persists for a comparatively long time, and is apparently due to the spontaneous segmentation of the annular rim at a very early stage; or, again, the very strongly salient ribs of flow just alluded to (see Series VI. of our first paper), each of which seems to correspond to one of the jets.

This general explanation is as follows:—

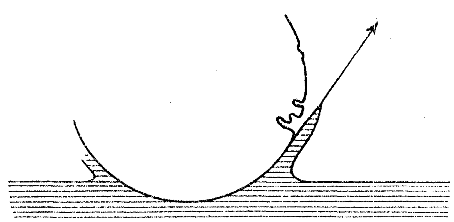
When a sphere, either rough or smooth, first strikes the liquid, there is an impulsive pressure between the two, and the column of liquid lying vertically below the elementary area of first contact is compressed. For very rapid displacements the liquid on account of its viscosity behaves like a solid. In the case of a solid rod we know that the head would be somewhat flattened out by a similar blow, and a wave of compression would travel down it; to this flattening or broadening out of the head of the column corresponds the great outward radial velocity, tangential to the surface, initiated in the liquid, of which we have abundant evidence in many of the photographs.

Into this outward flowing sheath the sphere descends, and since each successive zone of surface which enters is more nearly parallel to the direction of motion of the sphere, the displacement of liquid is most rapid at the lowest point, from the neighbourhood of which fresh liquid is supplied to flow along the surface. Whether the rising sheath shall leave the surface of the sphere or shall follow it depends upon the efficiency of the adhesion to the sphere. If the sphere is smooth, the molecular forces of cohesion will guide the nearest layers of the advancing edge of the sheath, and will thus cause the initial flow to be along the surface of the sphere.

To pull any portion of the advancing liquid out of its rectilineal path the sphere must have rigidity. If the advancing liquid meets loosely attached particles, *e.g.*, of dust, these will constitute places of departure from the surface of the sphere; the dust will be swept away by the momentum of the liquid which, being no longer in contact with the sphere, perseveres in its rectilineal motion. If the dust particles are few and far between, the cohesion of the neighbouring liquid will bring back the deserting parts, but if the places of departure are many, then the momentum of the deserters will prevail. Thus at every instant there is a struggle between the momentum of the advancing edge of the sheath and the cohesion of the sphere; the greater the height of fall the greater will be the momentum of the rising liquid, and the less likely is the cohesion to prevail, and the presence or absence of dust particles may determine the issue of the struggle.

Roughness of the surface will be equally efficient in causing the liquid to leave the sphere. For the momentum will readily carry the liquid past the mouth of any cavities (see fig. 3), into which it can only enter with a very sharp curvature of its

Fig. 3.



path. It is to be observed that the surface tension of the air-liquid surface of the sheath will act at all times in favour of the cohesion of the sphere, and even if the film has left the sphere the surface tension will tend to make it close in again, but we should not be right in attributing much importance to this capillary pressure which, with finite curvatures, is a force of a lower order of magnitude*

* This point may be made clearer by a numerical statement of the case. The particles of the liquid sheath on the shoulders of the sphere (itself *descending*) in figs. 5 and 6 of Series XIII., Plate 3, must be describing paths whose centres of curvature lie on the side of the sphere, and on every element of the film there must be an inwardly directed force. (We speak of the film as a whole, and ignore any minute vortical motion.) With a sphere of 1 centim. radius, and water as the liquid, the surface-tensional pressure would be about 0·075 gramme per sq. centim., which is quite insignificant as compared with the

than the cohesion, and, as later photographs will clearly show, is incompetent to produce the effects observed.

In order to test this general explanation further experiments were made.

Experiments on the Influence of Dust.

In the first place, to test the influence of dust, the experiment was made of deliberately dusting the surface of the sphere. For this purpose highly polished nickelled spheres, of the three sizes mentioned on p. 179, were held in a pair of crucible tongs by an electrified person standing on an insulating stool, and by him presented to any dusty object that stood or could be brought within reach. The particles of dust soon settled on the electrified sphere, which was then carefully placed on the dropping ring with the dusty side lowest. The liquid used was Alexandra oil, and the height of fall was 31·7 centims., at which each of these spheres when not dusted gave still a quite airless splash. When dusted an enormous bubble of air was carried down by each. Although the spheres when laid on the dropping ring must have completely lost their electrical charge, yet it seemed worth while to go through the same electrifying process without dusting them. The result showed that no change was produced. In order to see how far the influence of dust would go, the height of fall was now reduced, and it was found that with sphere (1) a fall of 17·1 centims. gave a perfectly rough splash when the surface was visibly dimmed with fine dust, and with sphere (3) a fall of 16·7 centims. availed. If the surface was only slightly dusty, then at these heights the splash remained "smooth."

It then occurred to us to try the effect of partial or local dusting, for we had already found by experimenting with a marked sphere that the method of dropping did not impart any appreciable rotation to the sphere, which reached the liquid in the attitude with which it started from the dropping ring. Accordingly, after dusting the sphere in the manner already described, the dust was carefully rubbed away from all but certain parts whose position was recorded. The experiments were very successful, and the results are shown in Series XX. The liquid used was water, and the sphere was of polished serpentine, 25·7 millims. in diameter, falling 14 centims. (*cf.* Series XIII., Plate 3).

In fig. 1 of Series XX. (see Plate 3) the sphere was dusted on the *right-hand*

atmospheric pressure on the outside, which would be about 1,033 grams per sq. centim. If the actual centripetal pull per unit area is less than this, then the hydrostatic pressure, even at the inner side of the film, will still be positive. If a greater pull than this is required, the hydrostatic pressure near the inner side of the film must be negative, and the liquid there will be in a state of true tension. Experiments which we have conducted *in vacuo*, and which will be described later, show that when this true tension is reached the liquid is *liable* to separate from the solid and to "cavitate," and the phenomenon of a smooth splash then ceases with the guiding influence of the sphere. Thus the limiting value of the cohesion which can be reached in practice is probably about 1,038 grams per sq. centim.

side and a "sound of splash" was recorded. On the left side there is no disturbance of the "smooth splash"; on the right is a "pocket" of air such as was obtained by accident in Series VI., fig. 4, of the earlier paper (here reproduced as fig. 2A for convenience of reference and to help the reader to interpret correctly the first and second figures of the present series). The point of departure at which the liquid left the sphere is well marked, and a tangent from this point passes through the outermost conspicuous droplets that must have been projected from it.

In fig. 2 the sphere was dusted *at the top and on the right-hand side, but not much more than halfway down,* and the configuration corresponds entirely to the facts. Here again a tangent from the well marked drops on the right-hand side leads very nearly to the place of departure from the surface of the sphere.

In fig. 3 on this page the record is that the sphere was dusty at the top only. It is to be expected that dusting at the top will not make *much* difference in the flow of the already converging liquid. Comparison with fig. 7 of Series XIII., Plate 3 shows, however, that a slight effect has been produced.

Series XX.

Fig. 3.

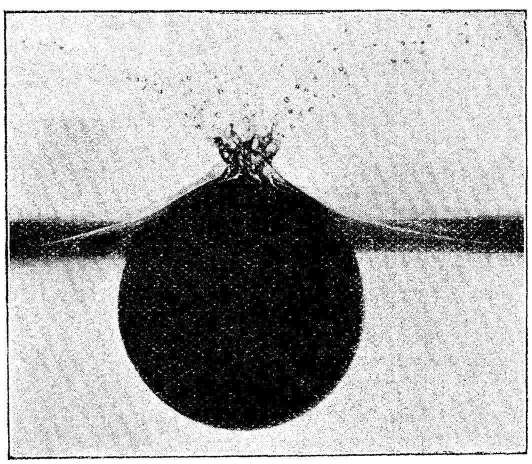
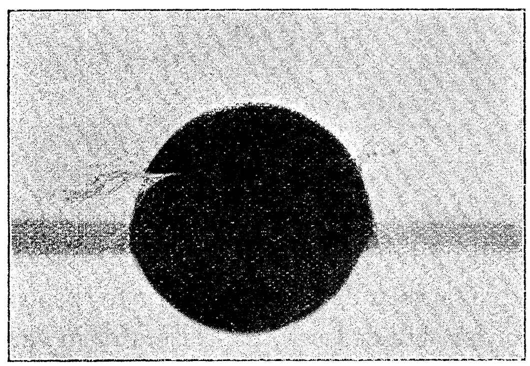


Fig. 4.



In fig. 4 the sphere was dusted at the bottom only. The appearance on the left-hand side seems to show that the liquid has, after leaving the sphere, again been brought within reach. This recovery at an early stage is explained by reference to photographs of Series X. of the splash of a rough sphere, which show that even the rough sphere is soon wetted for some distance up the sides, as we may imagine by the gradual passage of the sphere into the divergently flowing cone of liquid which surrounds the lower part. When the liquid again touches a polished part the film will be again guided up it in the manner already explained. In figs. 5 and 6 (shown in drawings on page 190) the sphere was out of focus through the slipping out of place of the rod which held the releasing gear—a fault which was not discovered till after the photographs had been developed. In each case the sphere was dusted on the right-hand side *along a narrow vertical strip.* In No. 5 the tangent from the highest drops on the right again leads accurately to the place of departure of the liquid. In fig. 6 the pocket of air has apparently been swept up the surface of the sphere, perhaps by the converging flow already noted.

Series XX.

Fig. 5.

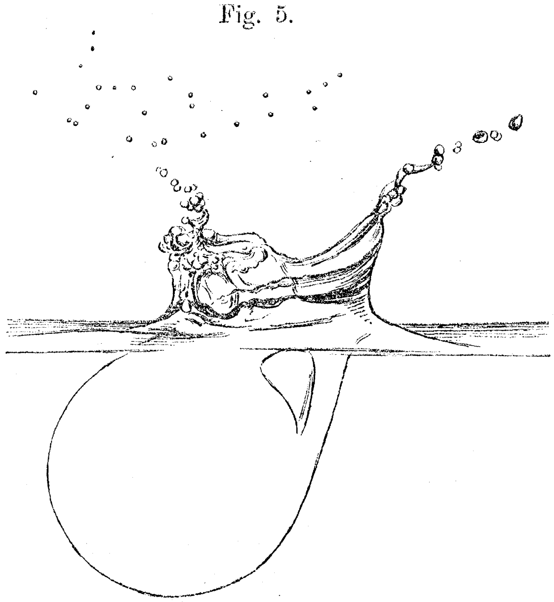
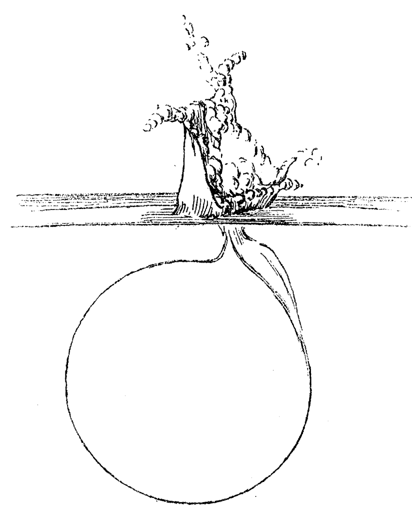


Fig. 6.



We observe that in figs. 1 and 2 (same Series XX., Plate 3) the continuous film or shell of liquid no longer reaches the outermost droplets that once have been at its edge. It must evidently have been pulled in by its own surface tension, which of course will cease to exercise any inward pull on a drop that has once separated.

The influence of dust, thus incontestably proved, seems to afford a satisfactory explanation of—

(1) The effect of a flame.
(2) The effect of heating.
(3) The variable and uncertain effects of electrification.

For (1) we may suppose that the flame burns off minute particles of dust; (2) we know from AITKEN'S experiments that dust from the atmosphere will not settle on a surface hotter than the air; (3) an electrified sphere descending through the air would attract dust to its surface unless it happened, as well might happen, that the air round about it, with its contained dust, had become itself similarly charged through the working of the electrical machine.

At the same time we cannot claim that our explanation of the influence of a flame is more than a conjecture. For we found that it was only when the brightly polished metal spheres were dusted nearly to dimness that the splash was invariably altered, but when we let such a visibly dusty sphere drop through a flame and then caught it in a conical wire cage, the dust was not found to be burned away or appreciably altered, *nor in this case was the splash altered by passage through the flame.* This shows that there is a kind of dust which cannot be removed by a flame, and it is only a conjecture, however probable, that there is a kind which can. We have sought to bring the matter to a crucial test by dropping the sphere through filtered and, presumably, dust-free air contained in a long wide iron pipe whose lower end was just submerged in water; but though the result of many such trials seemed to show that the splash near the critical height was more often "smooth" in the dust-free air than in ordinary air, there were too many exceptions for the matter to be put beyond doubt.

In further confirmation of our view that the leading clue to the explanation of the motion is the struggle between the adhesion of the rigid sphere and the tangential momentum of the liquid, we may cite the following points :—

A *liquid* sphere (see Series I., II., III. of Paper I.) makes a "rough" splash, and the photographs obtained show that the lower part of the in-falling drop is swept away by the tangential flow, while the upper part is still undistorted.

Here we have cohesion but no rigidity. And we have found that the "rough" splash is obtained by any process which gives a non-rigid surface to the sphere. Thus the splash made by a marble freshly roughened by sand-papering, or by grinding between two files and let fall from the very small height of 7·5 centims., can be practically controlled by attending to the condition of the surface. If the surface is quite dry and still covered with the fine powder resulting from the process of roughening, the splash is "rough," and a great bubble of air is taken down. But if this coat of powder, which has neither cohesion nor shearing strength, be removed by rubbing, the splash (under this low velocity) is "smooth." Again, a marble freshly sand-papered and covered with the resulting powder, if let fall from 12 or 15 centims., gives a rough splash. The same marble picked out of the liquid and very quickly dropped in again from the same height, will give again a rough splash. Here the liquid film is thick and "shearable." But if the same sphere be allowed to drain or be lightly wiped, the splash will be smooth. Here we may conjecture that enough fluid is left to fill up the interstices, but that the coat is not thick enough to shear easily. If, however, the sphere be thoroughly dried the splash becomes "rough" again. This gives us the explanation of the facts already recorded in respect of the splash of a wet sphere (Series VII. of Paper I.). This splash was always irregular; the liquid drifted to one side where it would shear, while it disappeared from the other or became there too thin to shear, though sufficient to fill up crevices.

Explanation of the Flutings.

The fact thus established experimentally, that the surface of a smooth sphere must be rigid if the film is to envelope it closely, suggests what seems a satisfactory explanation of the flutings. For it suggests that, even while the film is exceedingly thin, the flow is of the kind demanded by POISEUILLE, in which the liquid next to the solid has no motion relative to the solid, while the velocity farther away increases with the distance from the surface. Since the sphere is descending while the film is rising, there must be a strong viscous shear in the liquid impeding its rise. If by any fortuitous oscillation a radial rib arises, this will be a channel in which the liquid, being farther from the surface, will be less affected by the viscous drag; it will therefore be a channel of more rapid flow and diminished pressure, into which, therefore, the neighbouring liquid will be drawn from either side. Thus a rib once formed is in stable equilibrium, and will correspond to a jet at the edge of the rim. This explains the persistence of the ribs when once established, and we may attribute their regular distribution to the fact that they first originate in the spontaneous segmentation of the annular rim at the edge of the advancing sheath. This explanation receives unexpected confirmation in the appearance of the lop-sided splashes of Series XX., in which we see, firstly, that the flutings are absent from that part of the sheath which has left the sphere, and, secondly, we see how much higher in every case the continuous film has risen in that part which has left the sphere than in the part which has clung to it, and has been hindered by the viscous drag. Especially is this the case in fig. 3, Series XXVI. (see p. 196), where the liquid was pure glycerine. The effect of the viscous drag is, in fact, most marked in the most viscous liquid.

Influence of the Constants of the Liquid.

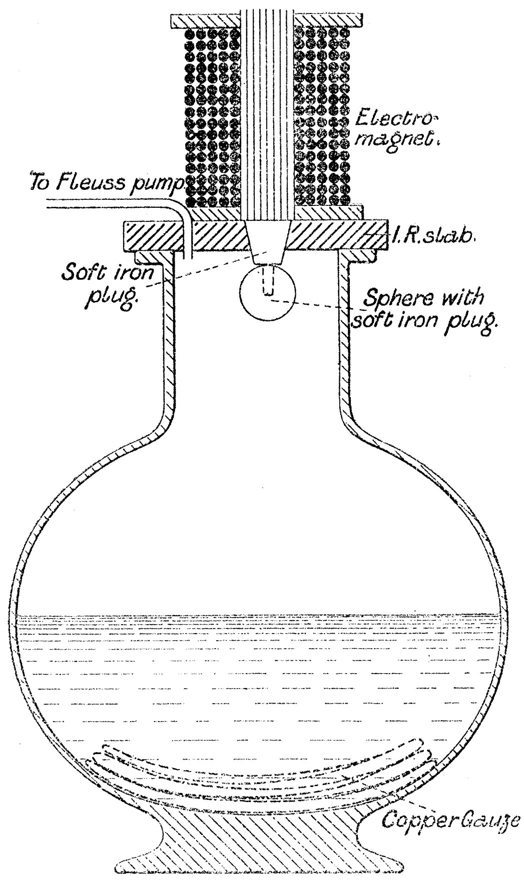
Finally, in confirmation of the general argument, we have the fact that with a liquid of small density and surface tension, such as Alexandra oil, a much smaller velocity of impact with a highly polished sphere is required to give "rough" splash than with water, a liquid of greater density and surface tension, the reason being without doubt that the tangential velocity due to the impact is greater with the less dense liquid, as, indeed, is proved to be the case by the greater height to which the surrounding sheath is thrown up, and the smaller the surface tension the less will be the abatement of velocity on account of work done in extending the surface.

[*Added January* 11, 1900.—The constants of the Alexandra oil employed in the experiments were as follows: *Specific Gravity*, 0·840; *Surface Tension*, 2·89 grams per metre (= surface tension of water × 0·383); viscosity = viscosity of water at the same temperature × 2·607. Experiments shortly to be described show that the addition to 51 vols. of water of as much as 6 vols. of glycerine, which must have largely increased the viscosity, produced little difference in the splash, except in making the ribs more noticeable, and we therefore conclude that within these limits the viscosity does not play an important part in determining the main course of the phenomenon.]

Experiments in vacuo.

It remained to examine what part was played by the air in the whole transaction. This could only be settled by removing the air. We accordingly made provision for obtaining, by instantaneous illumination, observations of splashes *in vacuo*. The method was simple enough, since, happily, very exact timing was not necessary. For eye observations a large, strong "bolt-head" was employed, and a 1-inch thick slab of india-rubber closed it air-tight (see fig. 4). This slab was pierced at one side by a glass tube leading to a Fleuss pump, and centrally by a short thick conical piece of soft iron, which served as the prolongation of the core of a straight electro-magnet which could be laid on the top. A pad of folded, fine woven, copper wire-gauze* prevented the bottom of the vessel being broken by the impact.

Fig. 4.



The nickelled and polished steel spheres were at first employed, but these retained so much of their magnetism that the timing was very uncertain, and they were after-

* We have found a pad of this material very convenient and efficient in all our experiments.

wards discarded for spheres of marble and serpentine, into which a deep hole was drilled, and into this a soft iron plug inserted. After each splash the air had to be re-admitted, the electro-magnet removed, and the vessel opened and the sphere fished out by means of a long, clean, bar-magnet.

The exhaustion was always pushed to within 2 or 3 millims. of a perfect vacuum, the vapour only of the liquid being left. The first observations were made in broad daylight, with a highly polished nickel sphere (size 1), dusted and undusted, and also with a rough marble sphere, 25·4 millims. in diameter. The observations were alternated by others, in which all conditions were the same except that the air was not removed. In no case could any difference be observed with the naked eye. Thus the polished sphere took down no air when clean, made a large bubble, or sometimes a "rough" column, when dusted, and the rough sphere always went in "rough" and made a high column. The depth of fall was 19·5 centims.

When spark-illumination was used with the small rough sphere, the figures [6], 8, and 10, and the upper part of [7] of the rough-sphere splash of Series X., Plate 2, were obtained repeatedly.

Even with the Fleuss pump, which works very quickly, it was a work of some minutes to exhaust this large bulb to within 1 or 2 millims. of a vacuum. During this time the electro-magnet had to be kept running, and became hot, and the time of de-magnetisation thus depended, more than was desirable, on the previous history of the magnetisation.

In order not to waste time and photographic plates, we exchanged this large vessel for a tall "gas jar" of smaller volume, through which the image was, indeed, a good deal distorted, but this does not much diminish the value of the record. We used both water and Alexandra oil as the liquids, and give, on Sheets 7 and 8, a few of the photographs thus obtained.

Inspection of these photographs shows only two points of difference of importance between the splash *in vacuo* and the splash in air.

The first of these is the significant point that it is not so easy to secure a quite smooth splash *in vacuo* for the reason, as we may confidently surmise, that the liquid, being supersaturated, is liable to burst into ebullition at the surface of the entering solid, where probably, as already explained in the note on p. 196, the velocities set up correspond to a true negative pressure or tension, under which the liquid will readily rupture and break away from the solid surface if any cavity is formed. Indeed, it should be mentioned that until the vessel had been many times exhausted of air, bubbles were very liable to form spontaneously in the liquid and rise from the bottom (boiling by bumping).

Fig. 1 of Series XXI., Plate 2, shows, for purposes of comparison, a smooth splash in air (height of fall, 14 centims.), while fig. 2, and also [3] and [4] (not here reproduced), show the near approach attained *in vacuo*, and the development of bubbles. Fig. [5] (not here given) shows a splash *in vacuo*, in which at an earlier stage the ensheathing film was almost complete. This photograph is practically identical with fig. 6, Series XIII., Plate 3.

Passing now to the "rough" splash, we have, in fig. 1, of Series XXII., Sheet 8 (see Plate 3), a "rough" splash in air; in fig. 2, the same *in vacuo;* in fig. [3], a later stage *in vacuo.* The liquid in each case was water, and the height of fall, 13·5 centims. Figs. [4] and [5] show the collapse of the vacuous column formed behind a rough sphere falling into Alexandra oil. In each of these last there is an appearance of folding at the surface, which was not observable when air was present. And we see the beginning of the same kind of thing on the left-hand side in fig. 2.

This difference, which is probably due to negative pressures induced by vortical motion near the interface, appears, however, to be quite a secondary matter, and does not prevent us from asserting that the presence of the air has no material influence on the early course of the splash, except, as already explained, in respect of the extent to which its pressure serves to relieve the liquid from a cohesive tension under which it would cavitate.

Experiments with a Viscid Liquid.

It appeared probable that experiments made with water thickened by successive additions of glycerine would throw light on the part played by viscosity in the transaction. With a mixture consisting of water 51 vols., glycerine 2 vols., no change of any kind was perceptible in the splashes observed. When the glycerine was increased to 6 vols. in 51 of water, the disturbance set up was still extremely similar, as will be seen by comparing the figures of Sheet 9, which represent the splash of a smooth serpentine sphere in the glycerine mixture at the heights specified, with the corresponding figures in the case of water, Sheet 4, Series XIV. and XVI. The only noticeable difference is the rather greater salience of the ribs in some of the glycerine figures, and the greater reluctance in the jets to segment into droplets.

We then experimented with pure glycerine. Series XXVI. of Sheet 10, reproduced in part on page 196, gives the splash in pure glycerine of the same polished serpentine sphere, 25·7 centims. in diameter, falling 75 centims. In all cases the radial ribs are seen in the negatives of the photographs to be very pronounced. Even at so early a stage as fig. 1 the fluting is well developed. The two photographs taken of stage 3 had each of them an isolated jet, probably owing to the fact that when working with so sticky a liquid it was difficult to avoid contaminating the cloth on which the sphere was each time re-polished after washing in water, with the result that the sphere behaved as if locally rough. The relatively great length of this jet brings out very well the part played by the viscous drag in hindering the flow of that portion of the liquid sheath which has remained in contact with the sphere. In the last figure No. 4, of this series the droplets just visible in the centre, below the level of the general surface, correspond to those of figs. 6 and 7 of Series XIII. at a much higher

Series XXVI.

Fig. 1.

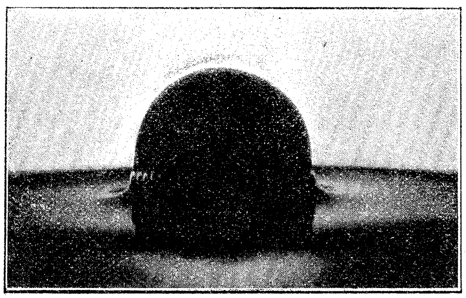


Fig. 3.

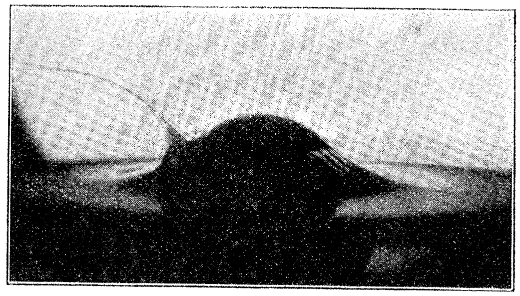
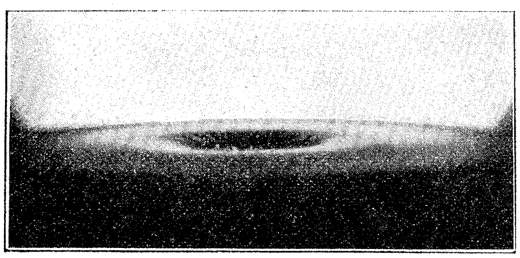


Fig. 4.



Series XXVII.

Fig. 1.

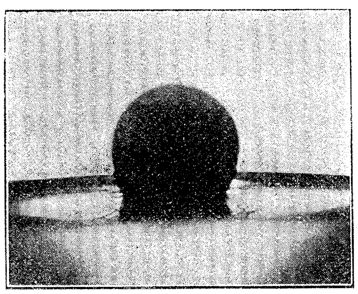


Fig. 2.

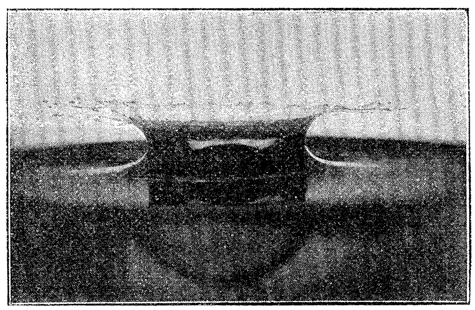
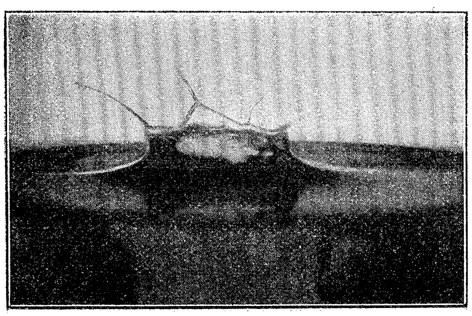


Fig. 3.

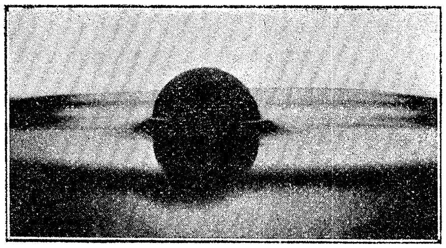


level, their presence in the lower position being again due to the slower convergence of the liquid sheath.

When we came to experiment with *rough* spheres falling 75 centims. into pure glycerine, the first photographs obtained were figs. 2 and 3 of Series XXVII., here given, which correspond very closely with figs. 2 and 3 of Series IX. of the former paper, obtained when a similar sphere fell 60 centims. into water. But when we adjusted the timing sphere so as to obtain earlier stages, expecting such a figure as No. 1 (which was actually taken from a water splash with 14 centims. fall), we obtained instead such figures as 1, 2, and 3 of Series XXVIII., in which the fall

Series XXVIII.

Fig. 1.



Fig, 2.

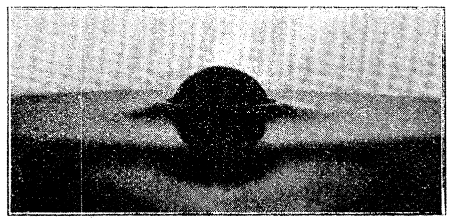


Fig. 3.

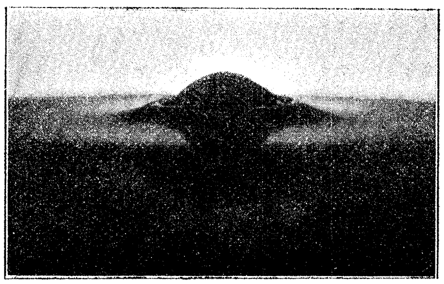


Fig. 4.

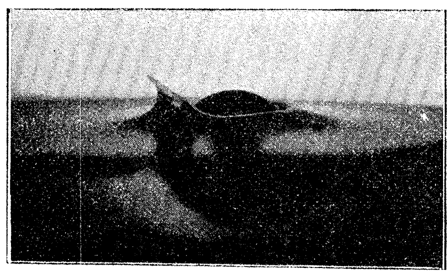
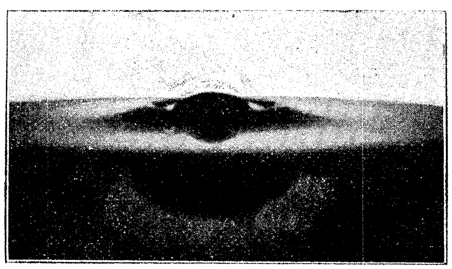


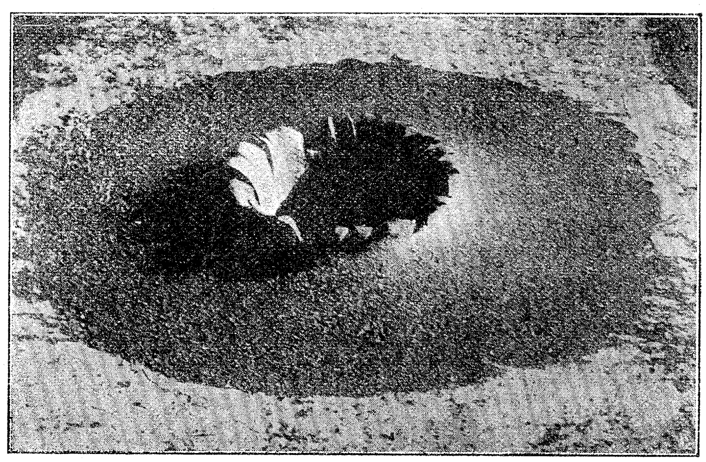
Fig. 5.



was the same, viz., 75 centims. Being convinced that the extreme fringe of the crater in fig. 2, Series XXVII., could only have been projected at a very early stage, we made repeated experiments to discover the cause of the contradiction, and found that such figures as 2, and 3, and 4 of Series XXVII. were only obtained after the glycerine had stood long enough for the exposed surface to absorb a film of water from the air. Thus, if the glycerine was freshly stirred, Series XXVIII. was invariably obtained, but if it stood for twelve hours exposed to the air of the laboratory the splash was that of Series XXVII. We found that the gain of weight was about 0·01 gramme per sq. centim. of exposed surface in twenty-four hours, so that the water absorbed in twelve hours would, if it remained on the surface, form a layer about $\frac{1}{20}$ of a millim. thick. Using a fall of 204 centims., we found by naked eye observation that the water absorbed in six hours did not suffice to change the splash, while that absorbed in twelve hours always sufficed. These observations throw a striking light on the determining importance of the initial motion. It should be mentioned that in Series XXVII. the temperature of the glycerine was 15° C., and in Series XXVIII. was 12° C., except in the last figure, when the fall was 100 centims. and the temperature 22°.

Further experiments with viscous liquids are very desirable, as they may enable us to pass by gradual transition to phenomena which at first sight may appear to be far removed. For if any one will compare with fig. 2 of Series XXVII., the accompanying photograph of the permanent record left of the splash which a steel projectile makes on entering a hard steel armour plate on the entering side, he will find it difficult to resist the belief that the plate has behaved like a liquid. Yet even the

Fig. 5.



whole kinetic energy at disposal in such an impact would not suffice to raise the projectile itself through more than a few hundred degrees Fahrenheit, still less to melt at the same instant any appreciable quantity of metal, and we are therefore driven to the conclusion that under the enormous pressure due to the impact the physical properties of the material of the plate have been so far altered as to change entirely the conditions of liquefaction.

Series XII.

Fig. 1.

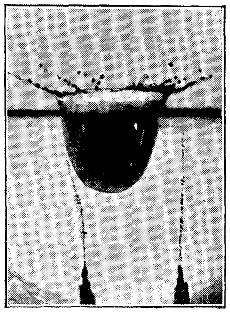


Fig. 2.

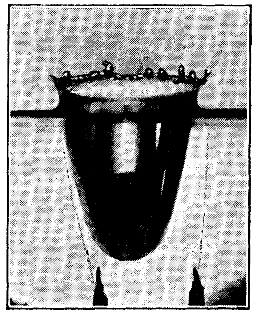


Fig. 3.

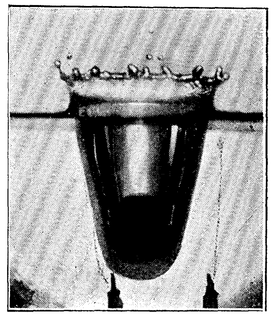


Fig. 5.

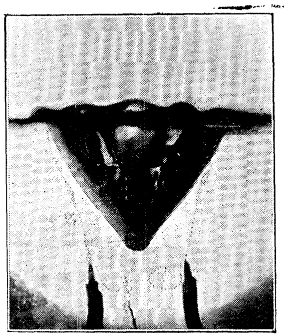
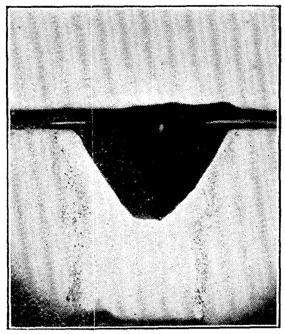
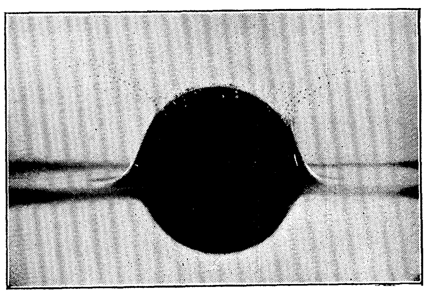


Fig. 6.



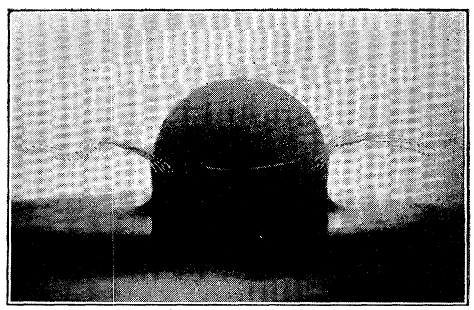
Series XIV.

Fig. 1.



Series XV.

Fig. 2.



Series XVII.

Fig. 1.

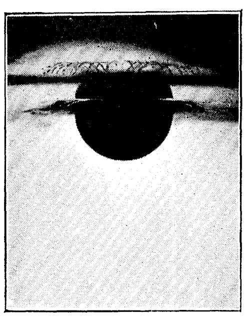


Fig. 2.

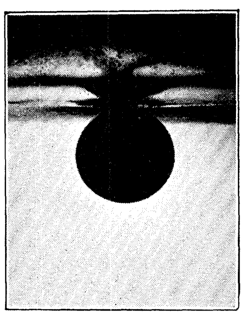


Fig. 3.

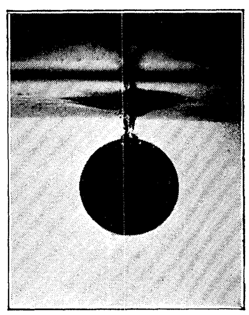
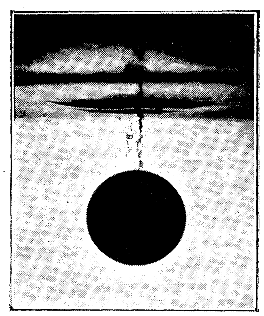


Fig. 4.



Series XVIII.

Fig. 1.

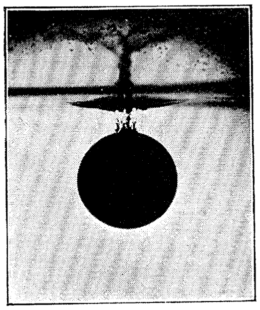


Fig. 2.

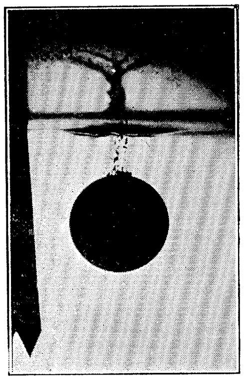


Fig. 3.

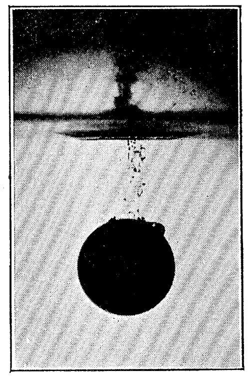
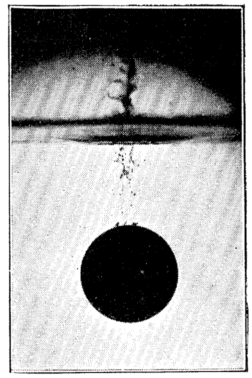


Fig. 4.



Series XIX.

Fig. 1.

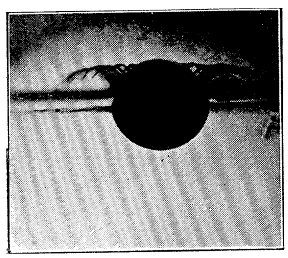


Fig. 2.

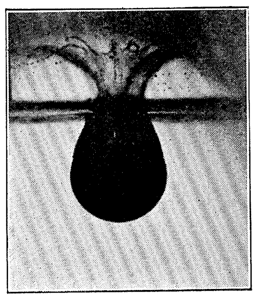


Fig. 3.

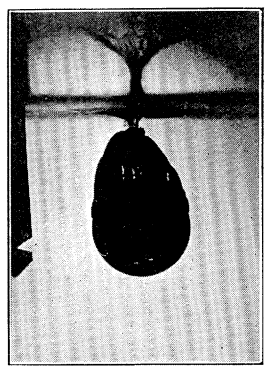


Fig. 4.

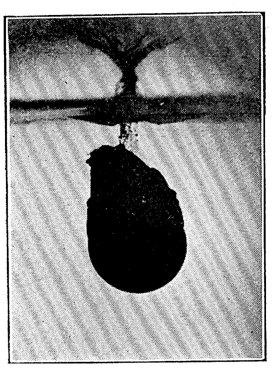


Fig. 5.

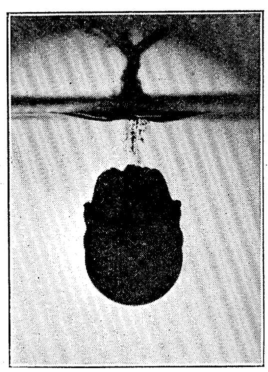


Fig. 6.

Series XX.

Fig. 3.

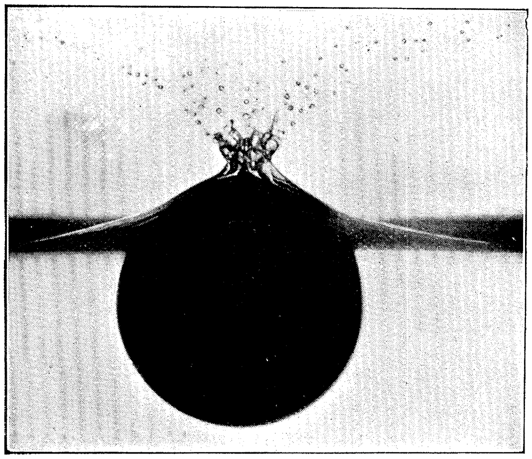
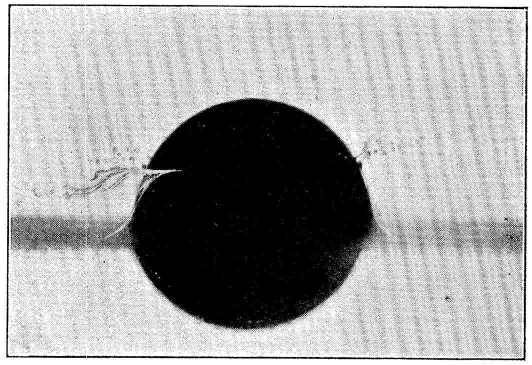


Fig. 4.



Series XXVI.

Fig. 1.

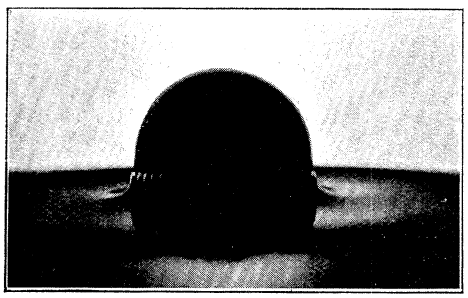


Fig. 3.

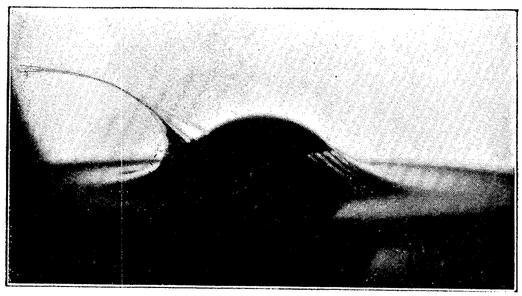
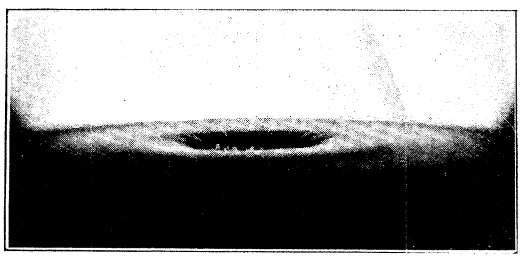


Fig. 4.



Series XXVII.

Fig. 1.

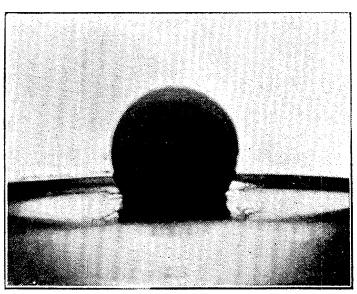
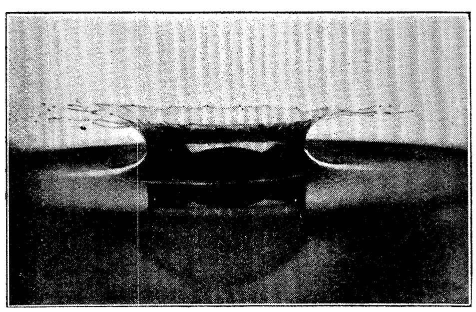
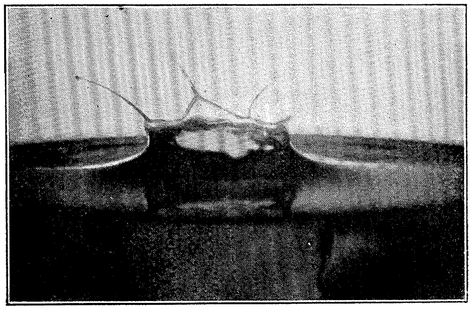


Fig. 2.



Series XXVII.

Fig. 3.



Series XXVIII.

Fig. 1.

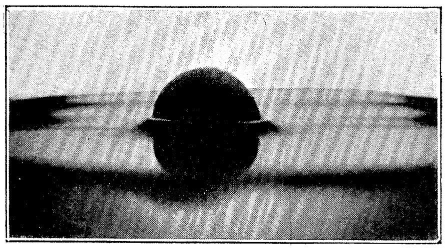


Fig. 2.

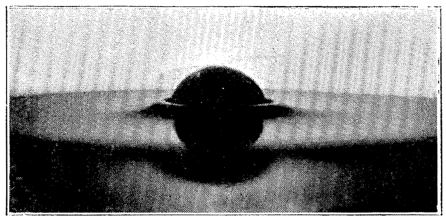


Fig. 3.

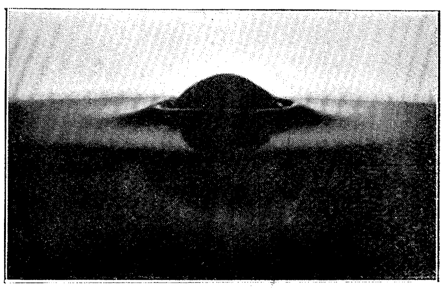


Fig. 4.

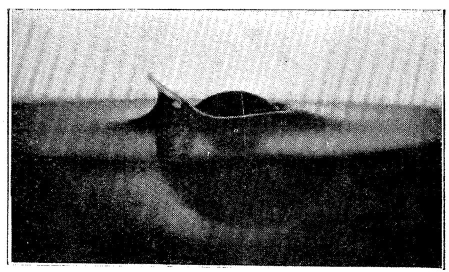


Fig. 5.

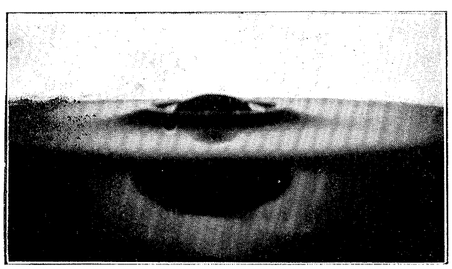
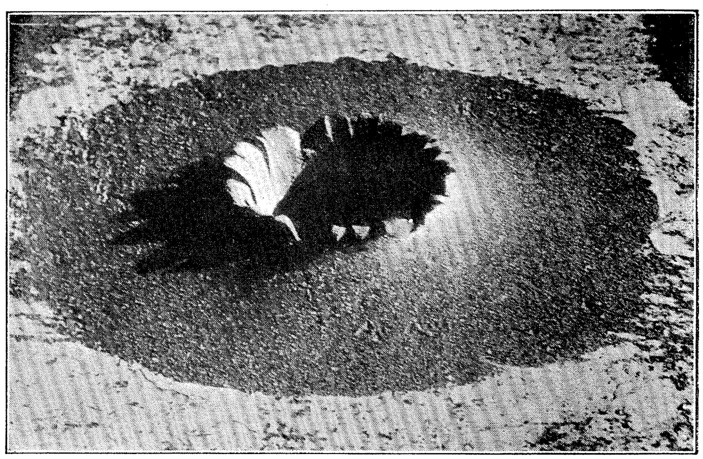


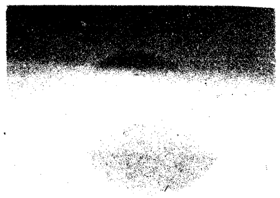
Fig. 5.



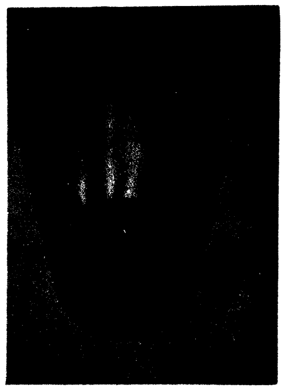
Such a conclusion, if established, would have practical importance in many mechanical processes. It could probably be tested by a microscopic examination* of specimens of the metal of the plate taken from the immediate neighbourhood of the "splash," and it is interesting to recall in this connection the argument of Professor POYNTING in his paper on the Change of State: Solid—Liquid ('Phil. Mag.,' July, 1881), that the rate of exchange of molecules across any surface will increase with the pressure.

* Since this was written Sir WM. ROBERTS-AUSTEN has most kindly examined for us specimens of the metal taken from the "burr" of such an armour-plate splash, and reports that he finds no traces of liquefaction having occurred. It therefore appears that, even in this extremely rapid deformation, we may have to attribute the plasticity and quasi-fluidity of the metal to the same slip along surfaces of cleavage within the crystals of the material, which Professor EWING and Mr. ROSENHAIN† have shown to take place when the deformation is much more slowly effected.—(Note, October 8, 1899.)

† 'Roy. Soc. Proc.,' May 25, 1899, vol. 65; 'Phil. Trans. R.S.,' Series A, vol. 193, 1899.



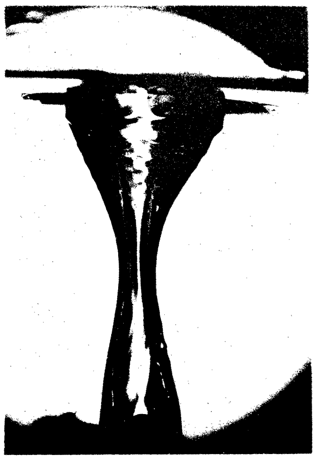
1



5

8

9



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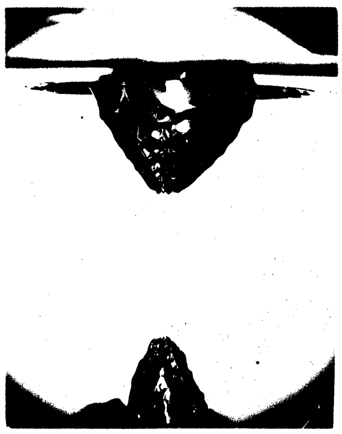

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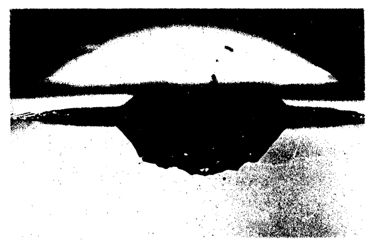


13

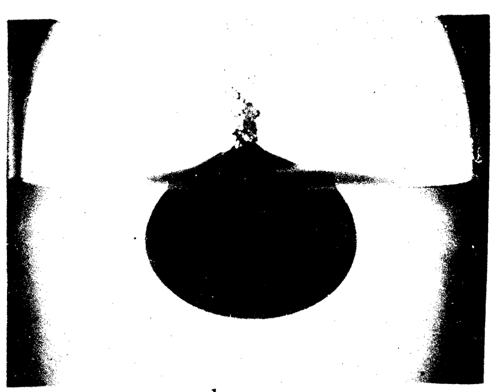
16

SERIES X.

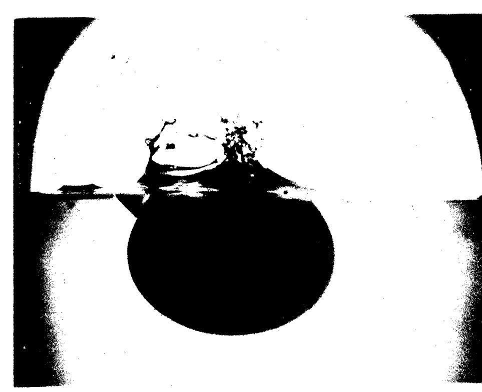
15

17

SERIES XXI.



1



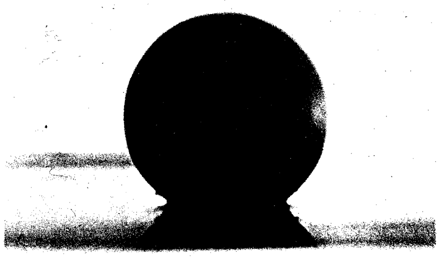
2

SERIES XXII.

1

2

SERIES XIII. (15 cm. fall).



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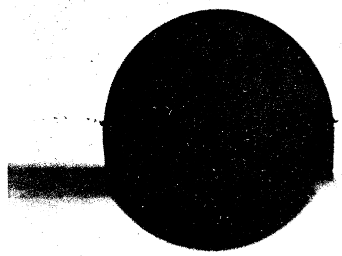

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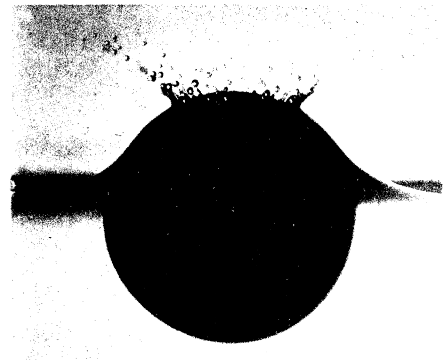

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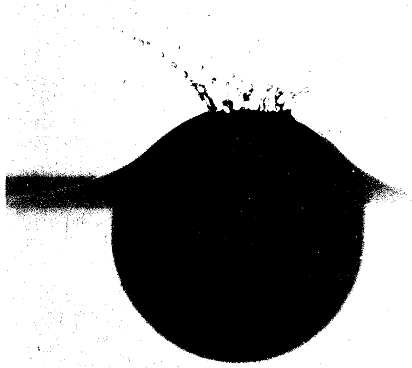

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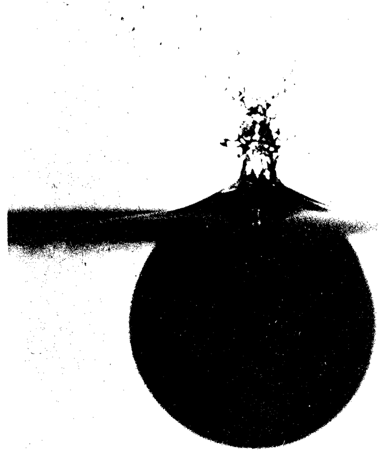
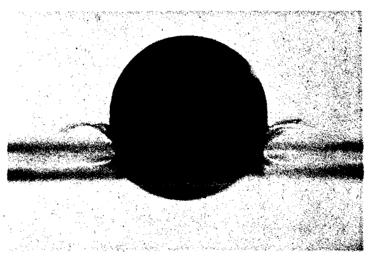


7

8

SERIES XVI. (100 cm. fall).



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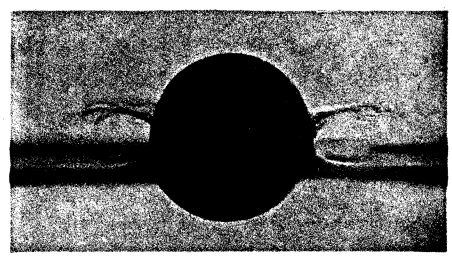

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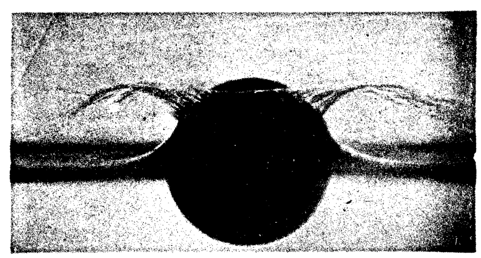


3

1

2A.

SERIES XX.

2

Series XII.

Fig. 1.

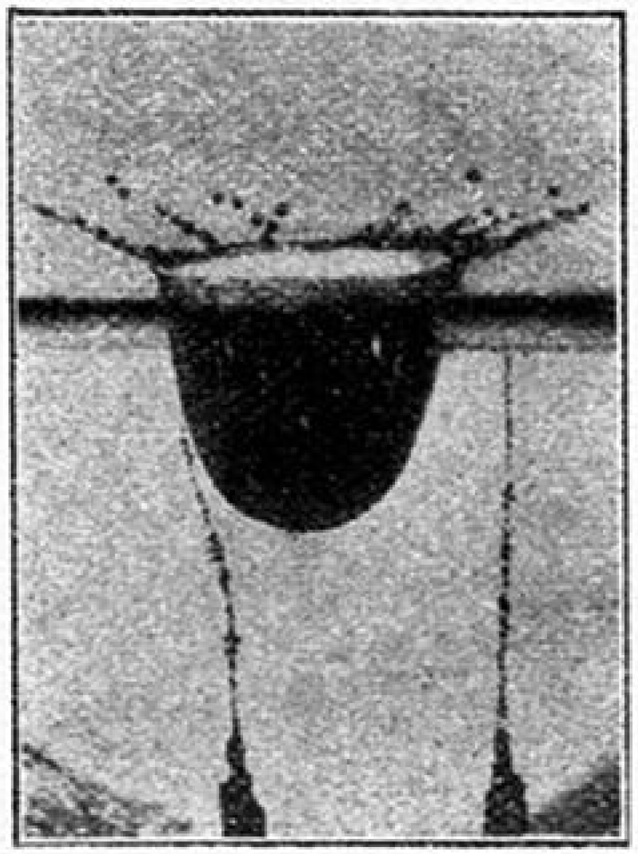


Fig. 2.

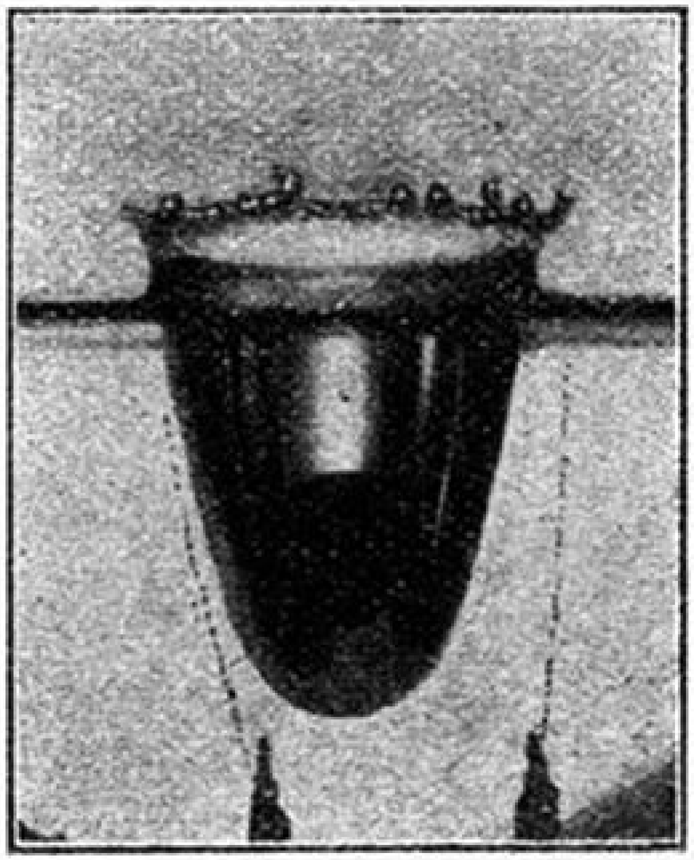


Fig. 3.

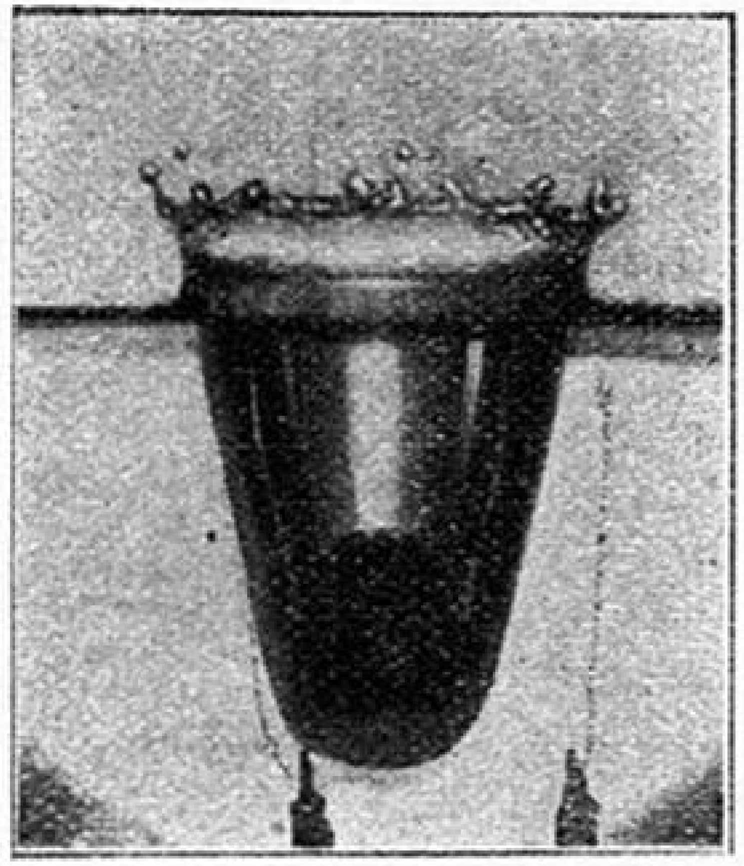


Fig. 5.

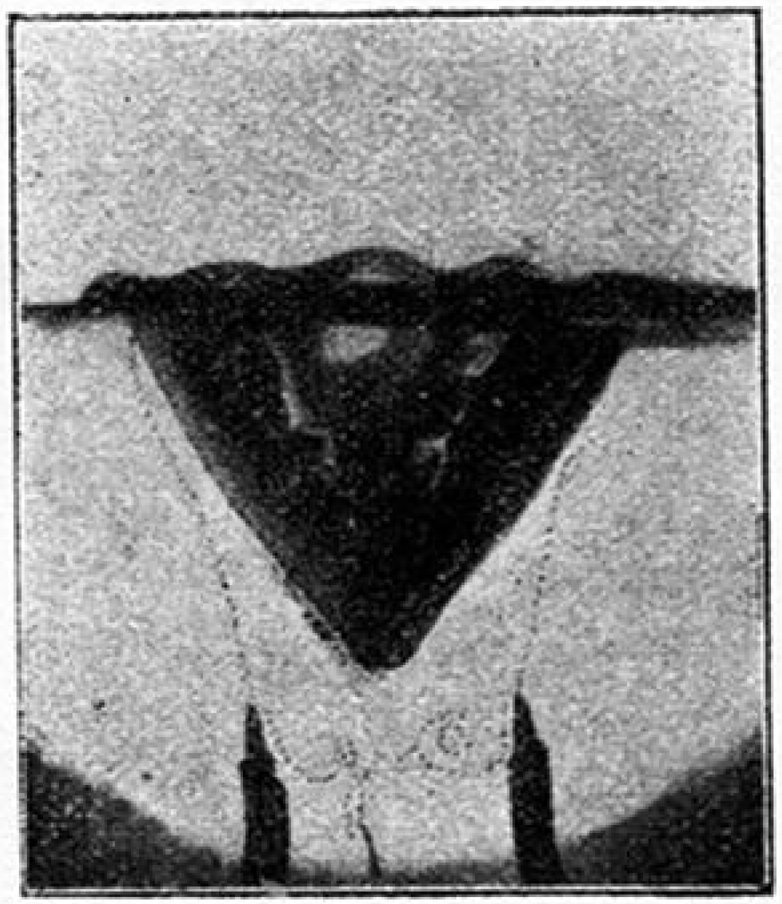
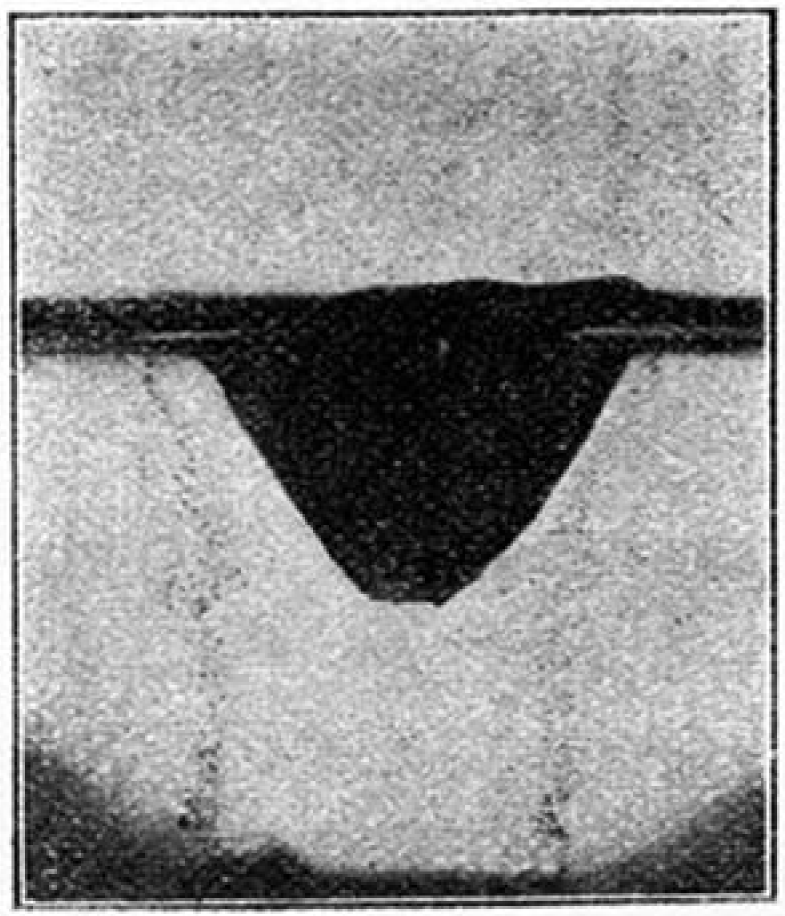
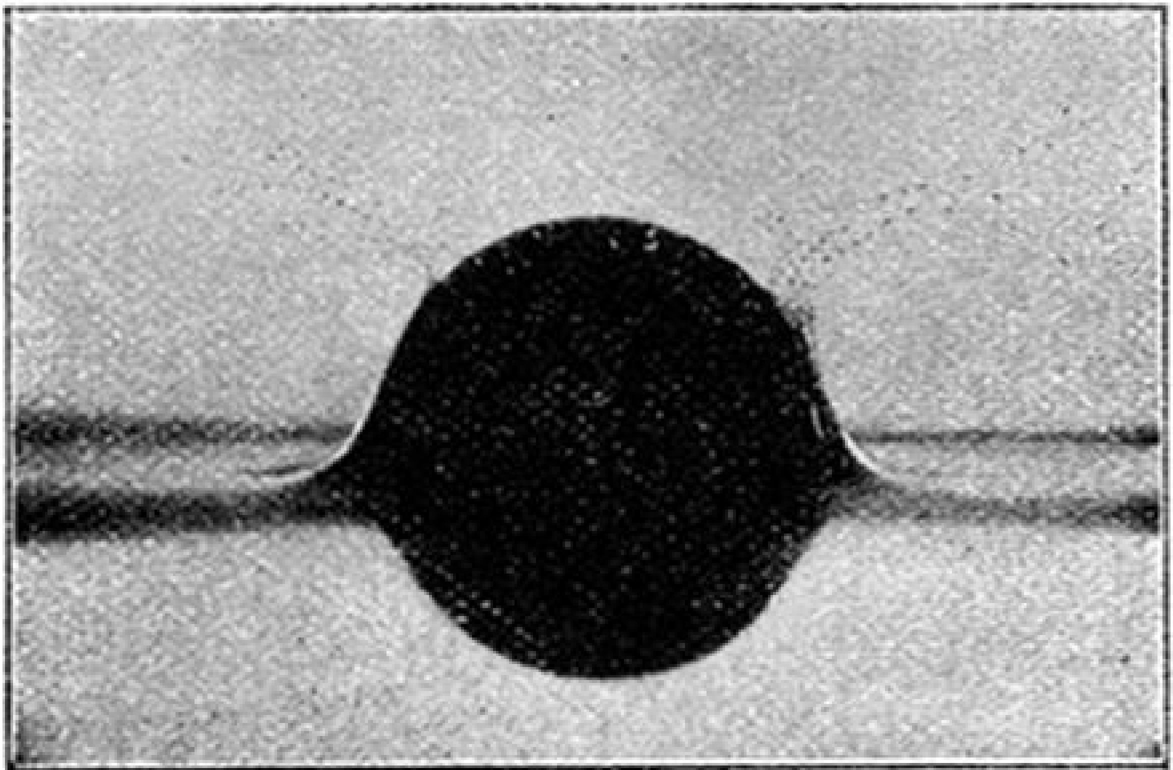


Fig. 6.



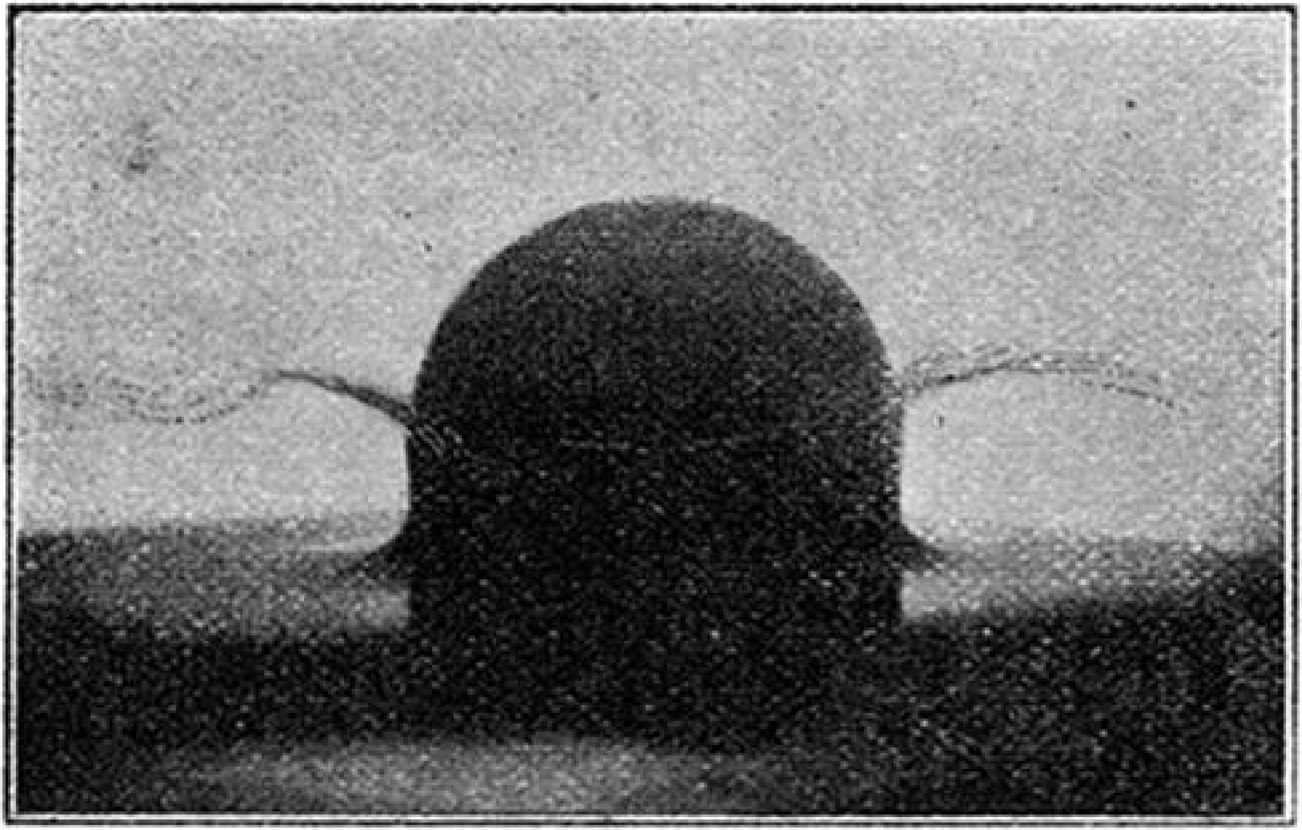
Series XIV.

Fig. 1.



Series XV.

Fig. 2.



Series XVII.

Fig. 1.

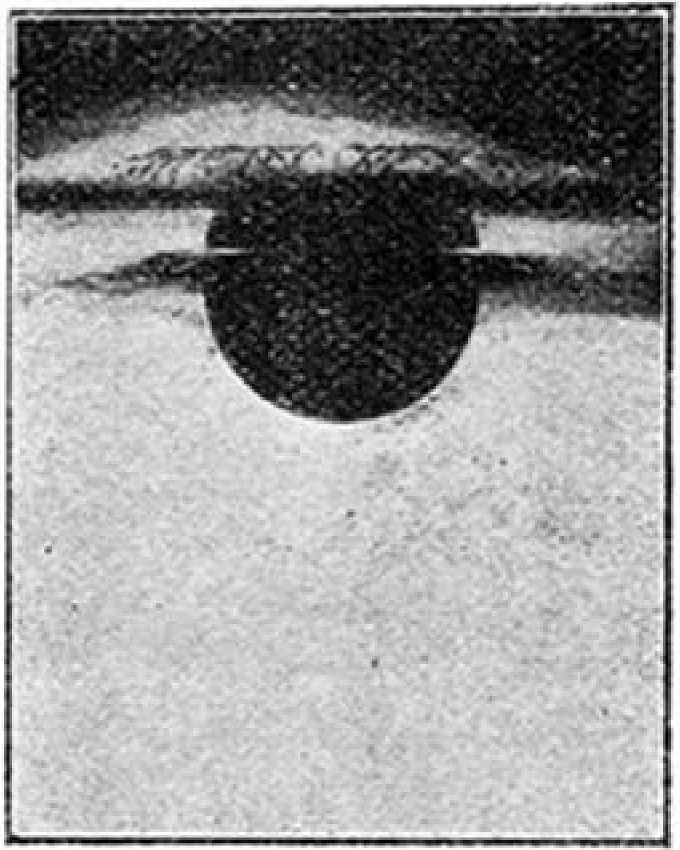
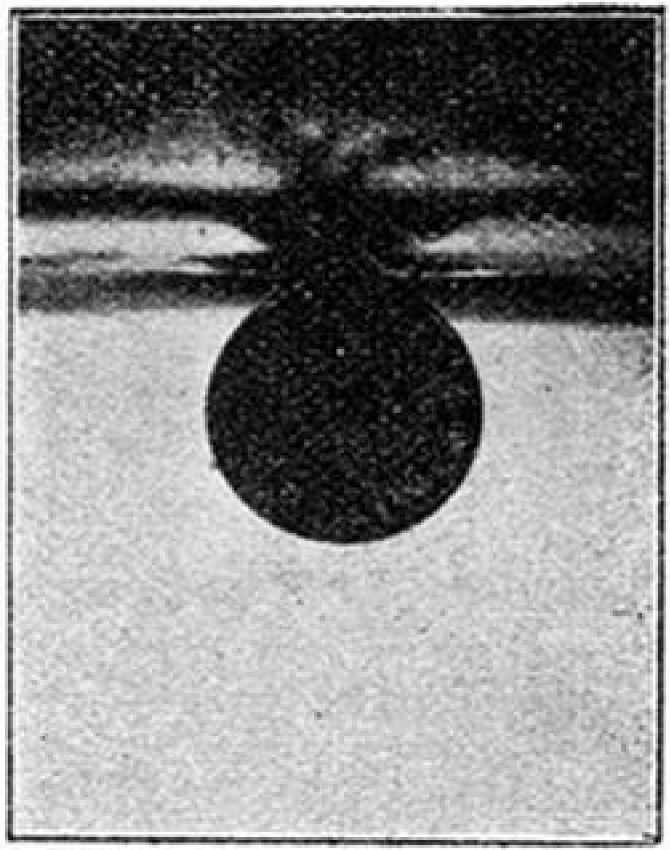


Fig. 2.



Series XVII.

Fig. 3.

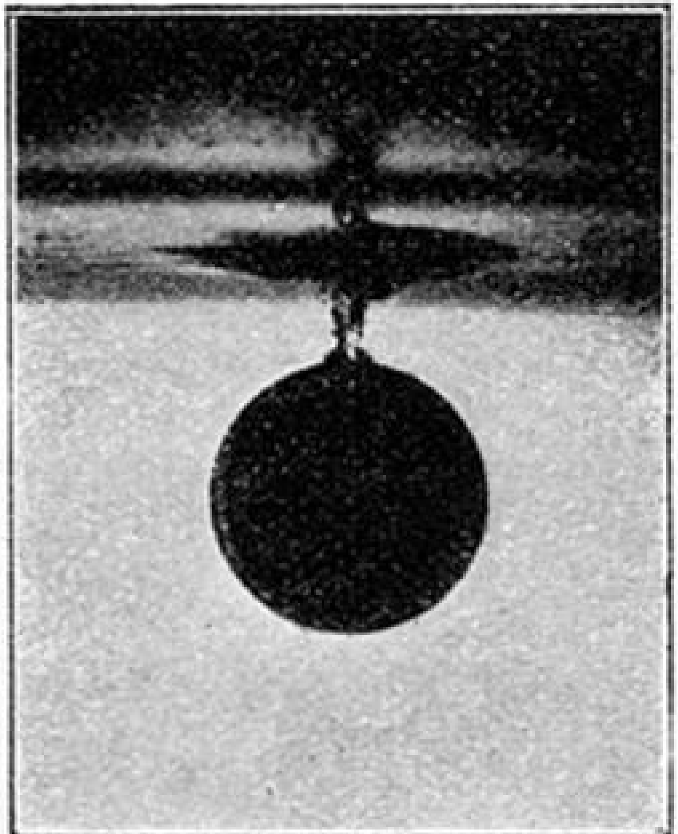
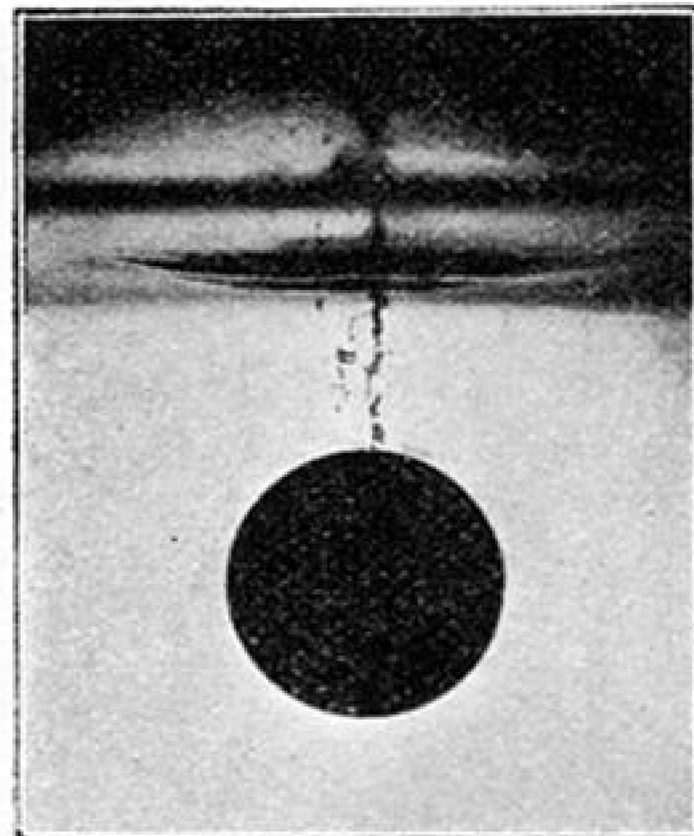


Fig. 4.



Series XVIII.

Fig. 1.

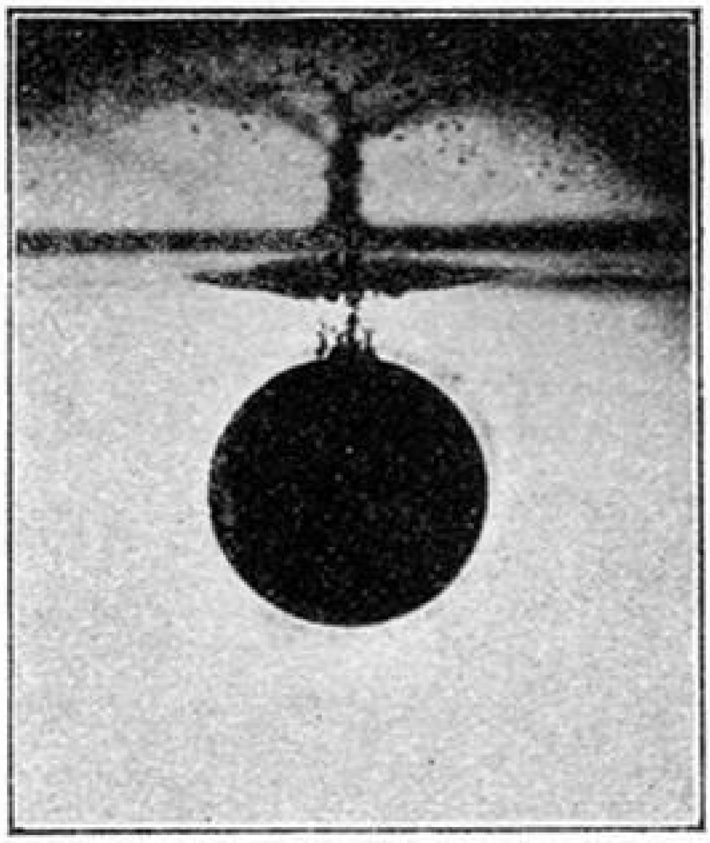


Fig. 2.

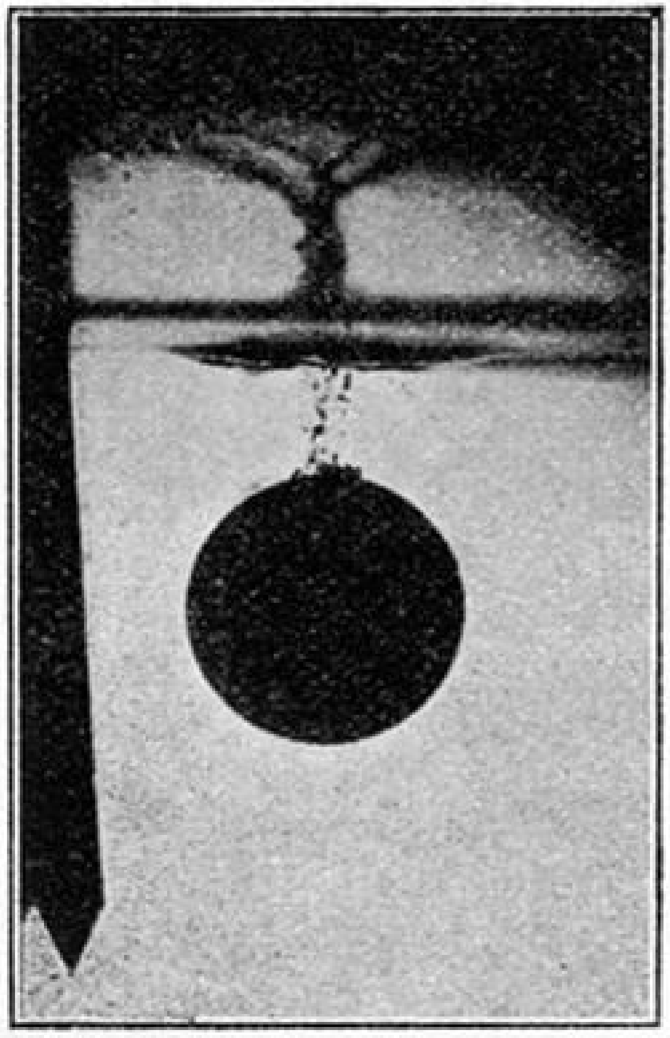


Fig. 3.

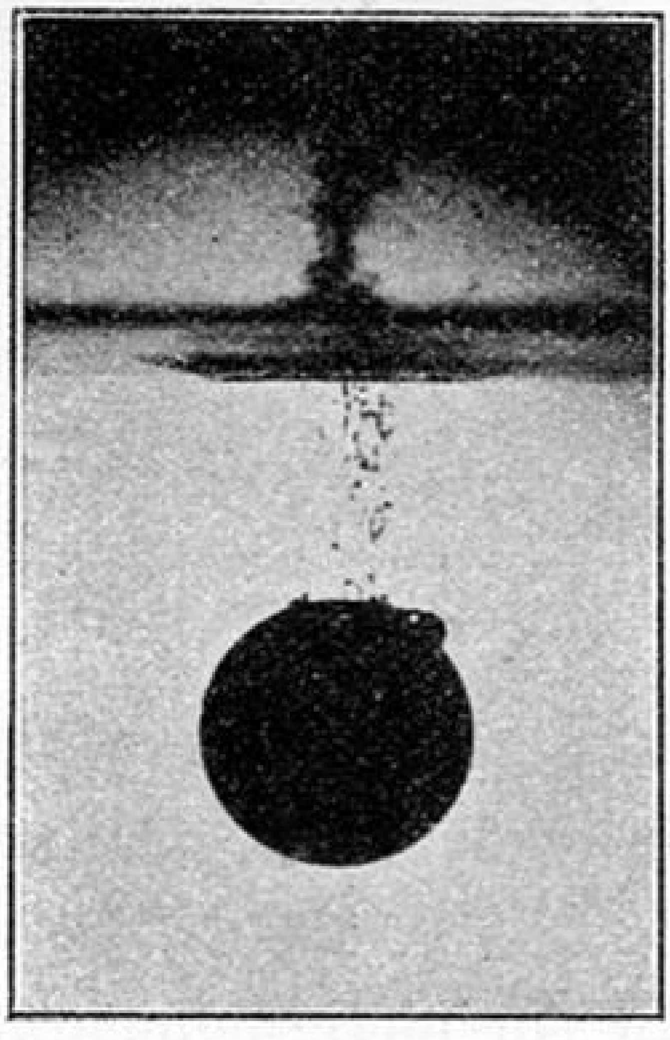
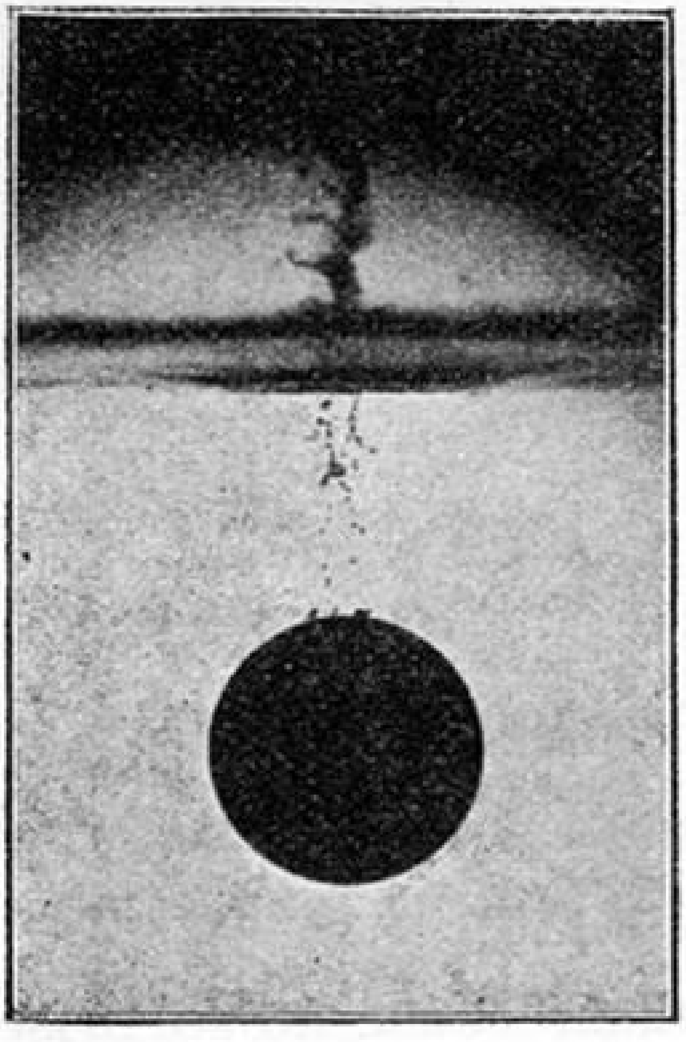


Fig. 4.



Series XIX.

Fig. 1.

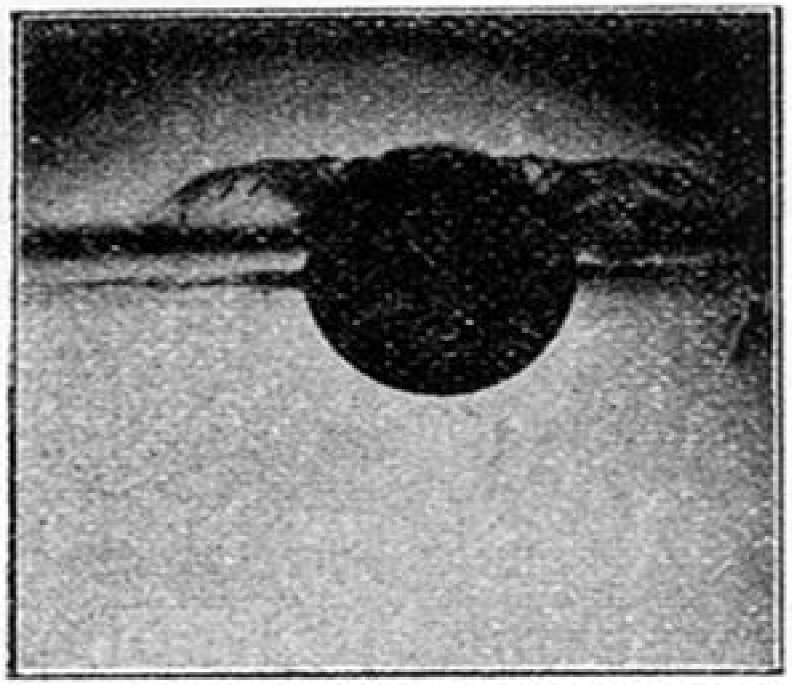


Fig. 2.

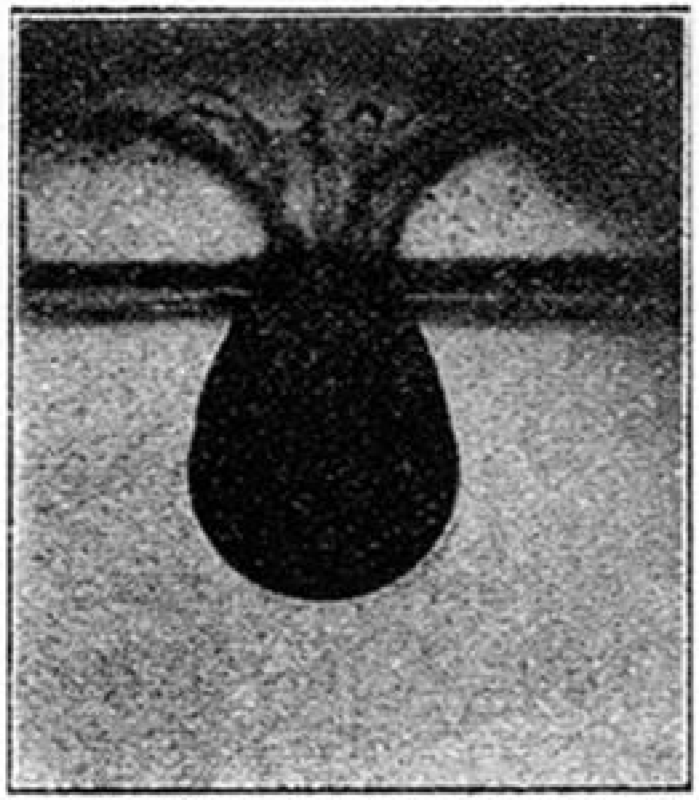


Fig. 3.

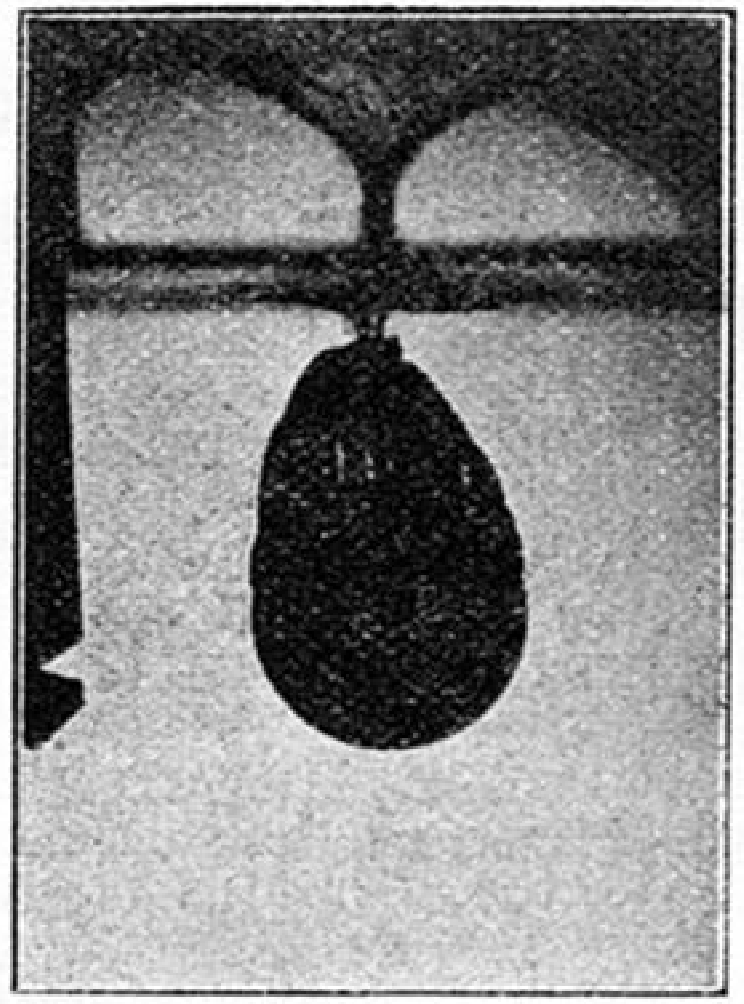


Fig. 4.

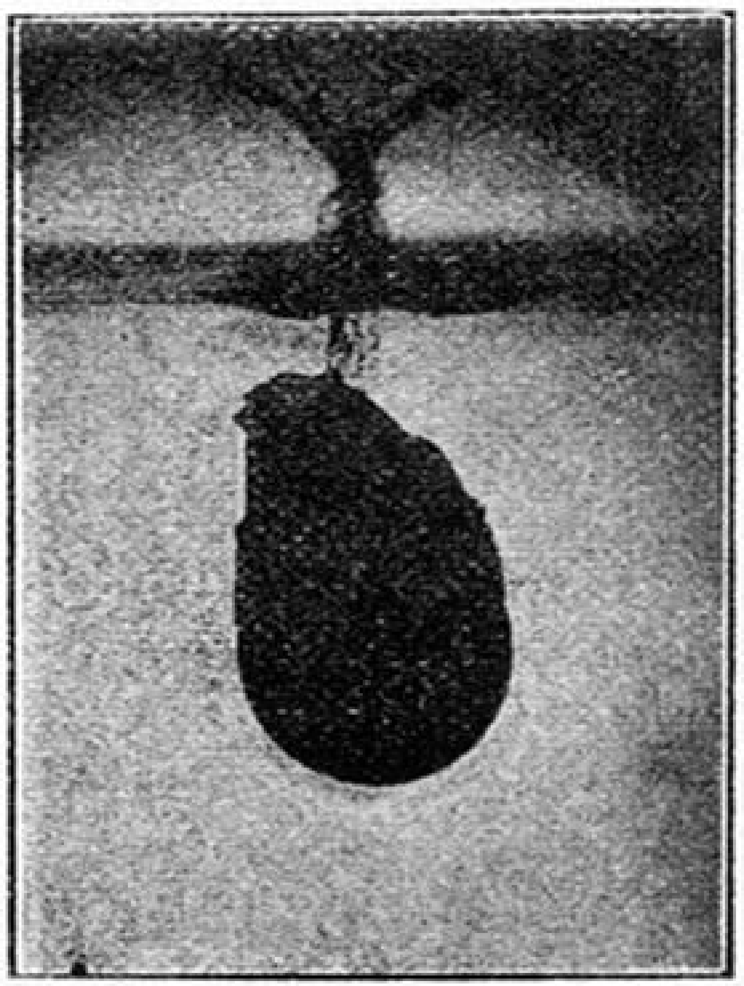


Fig. 5.

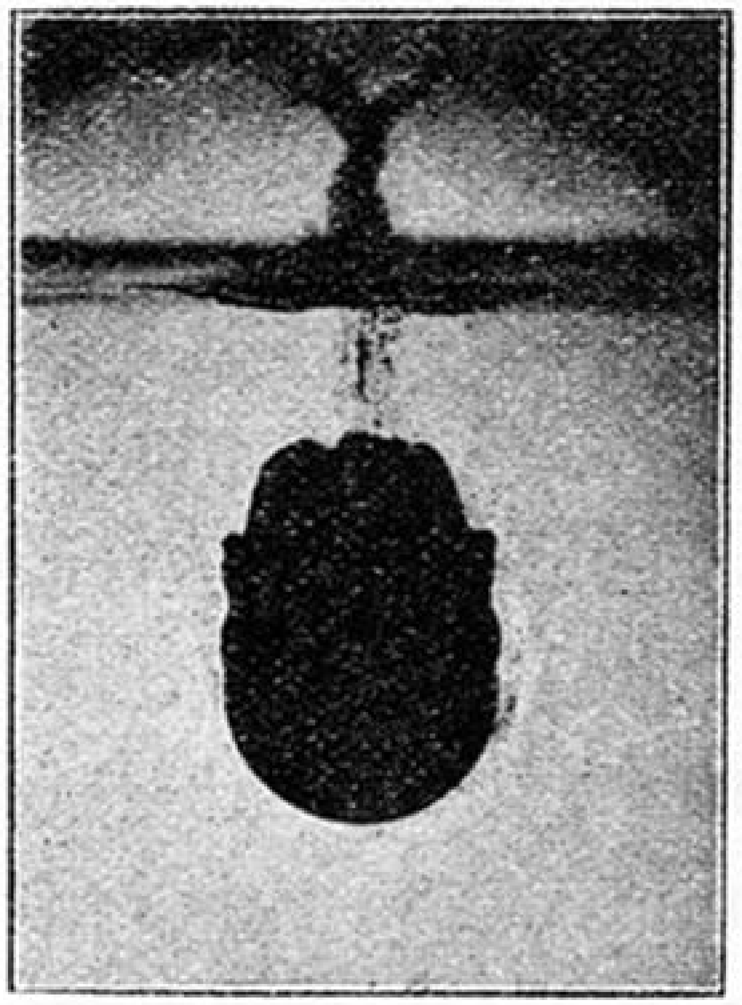
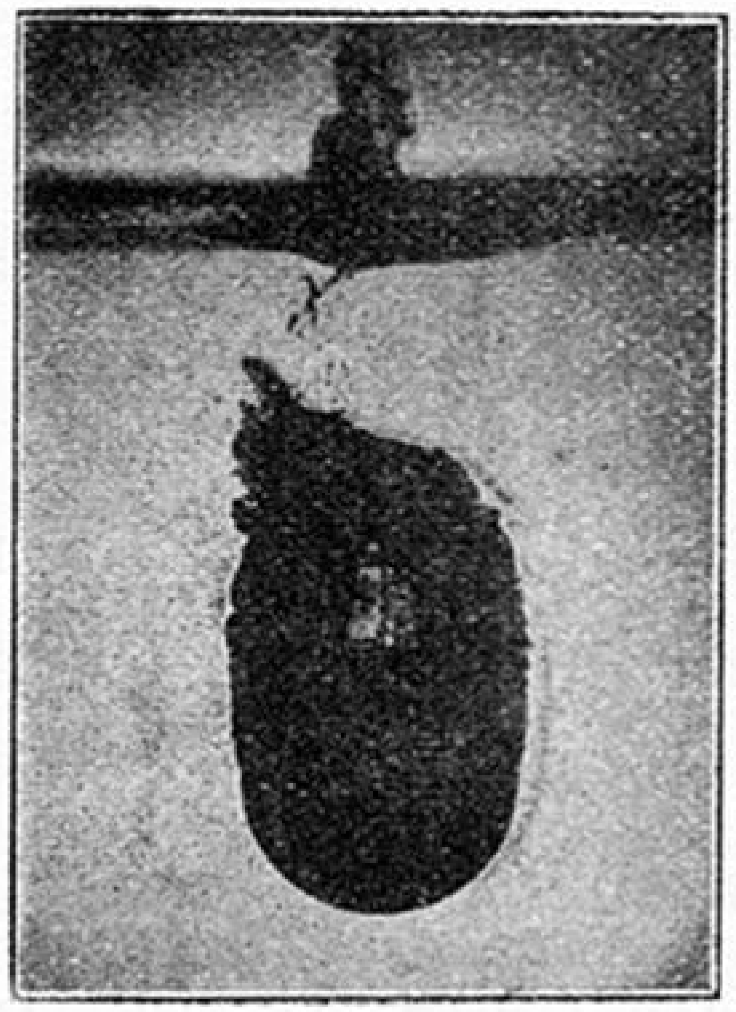


Fig. 6.



Series XX.

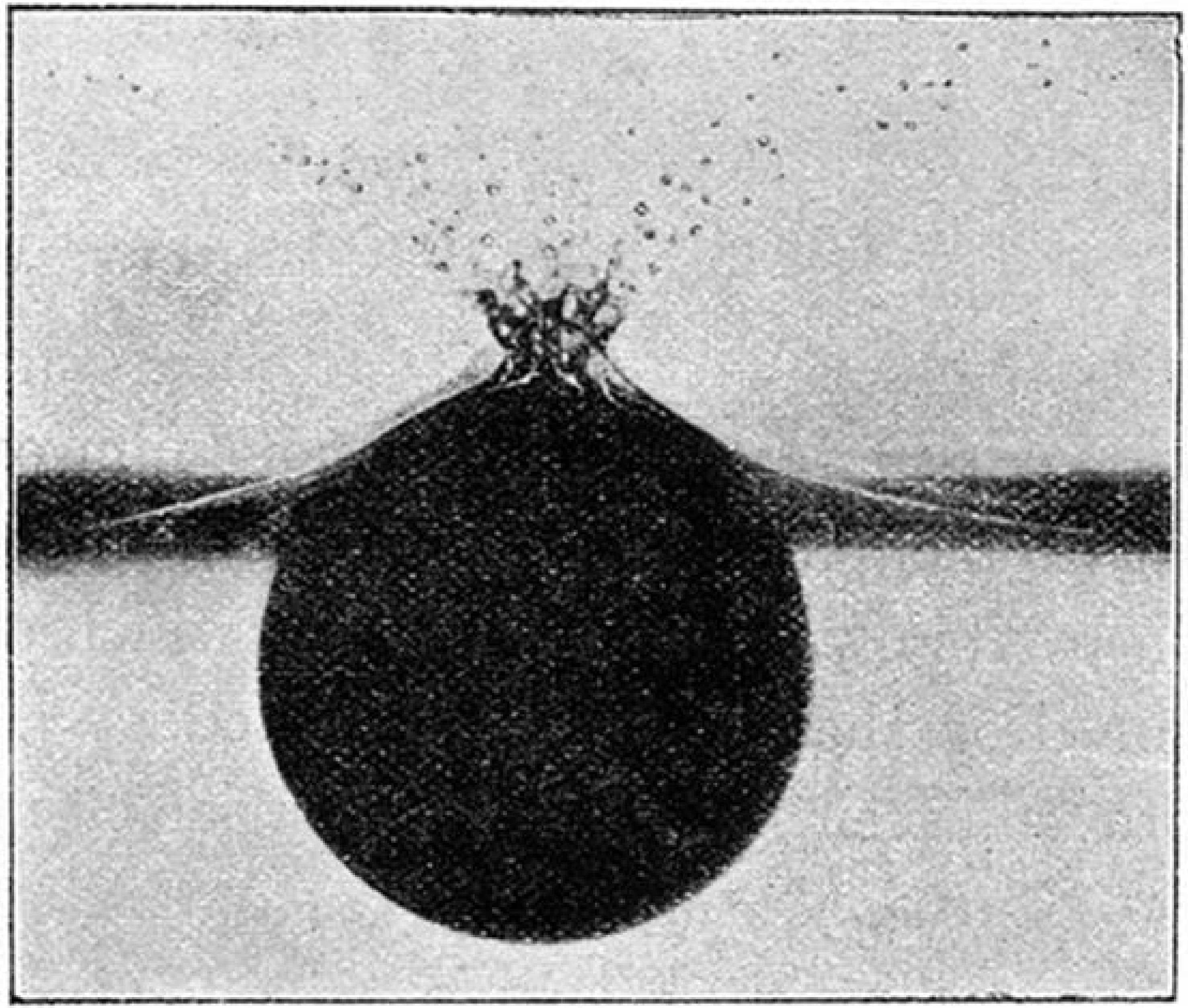


Fig. 3.

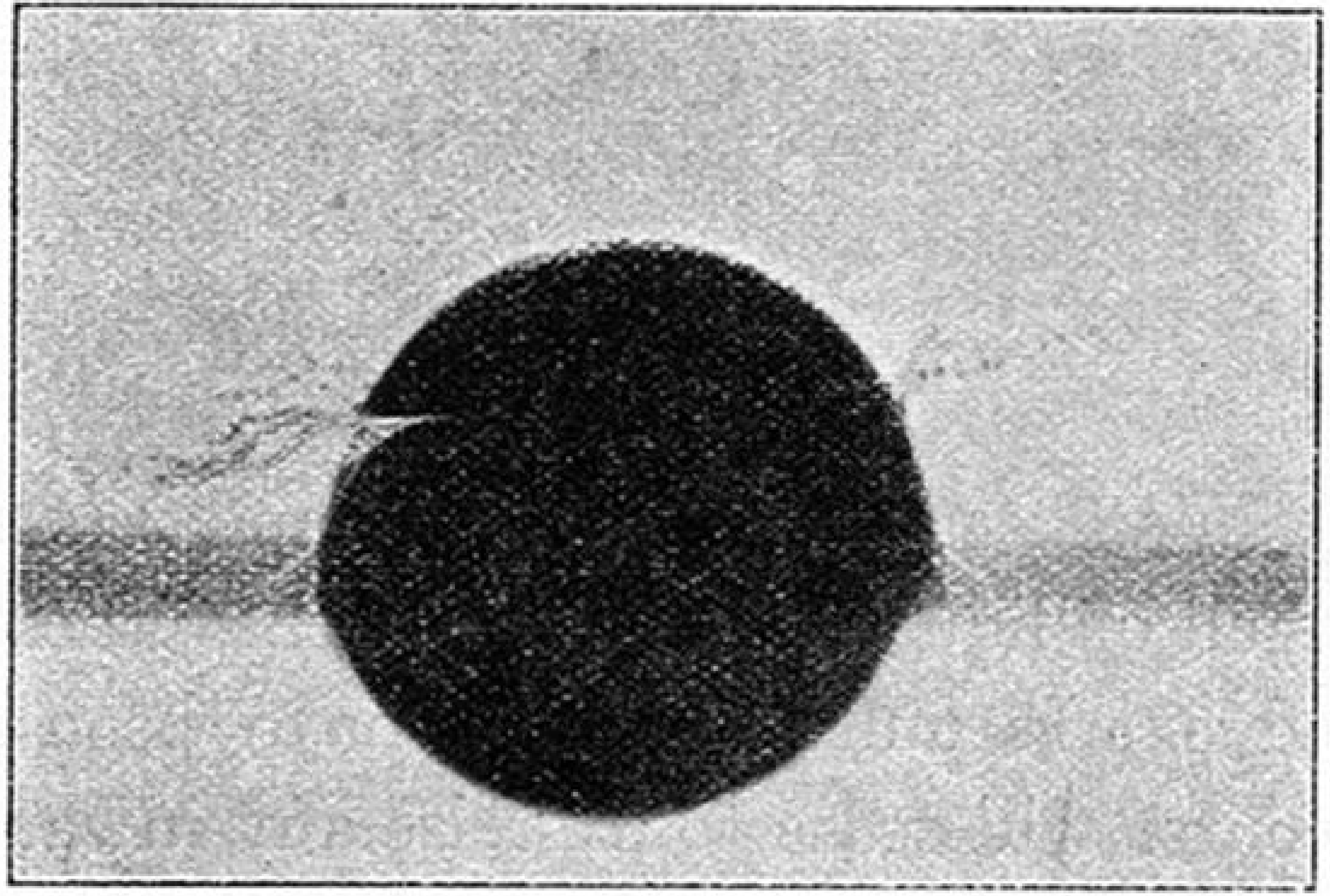


Fig. 4.

Series XXVI.

Fig. 1.

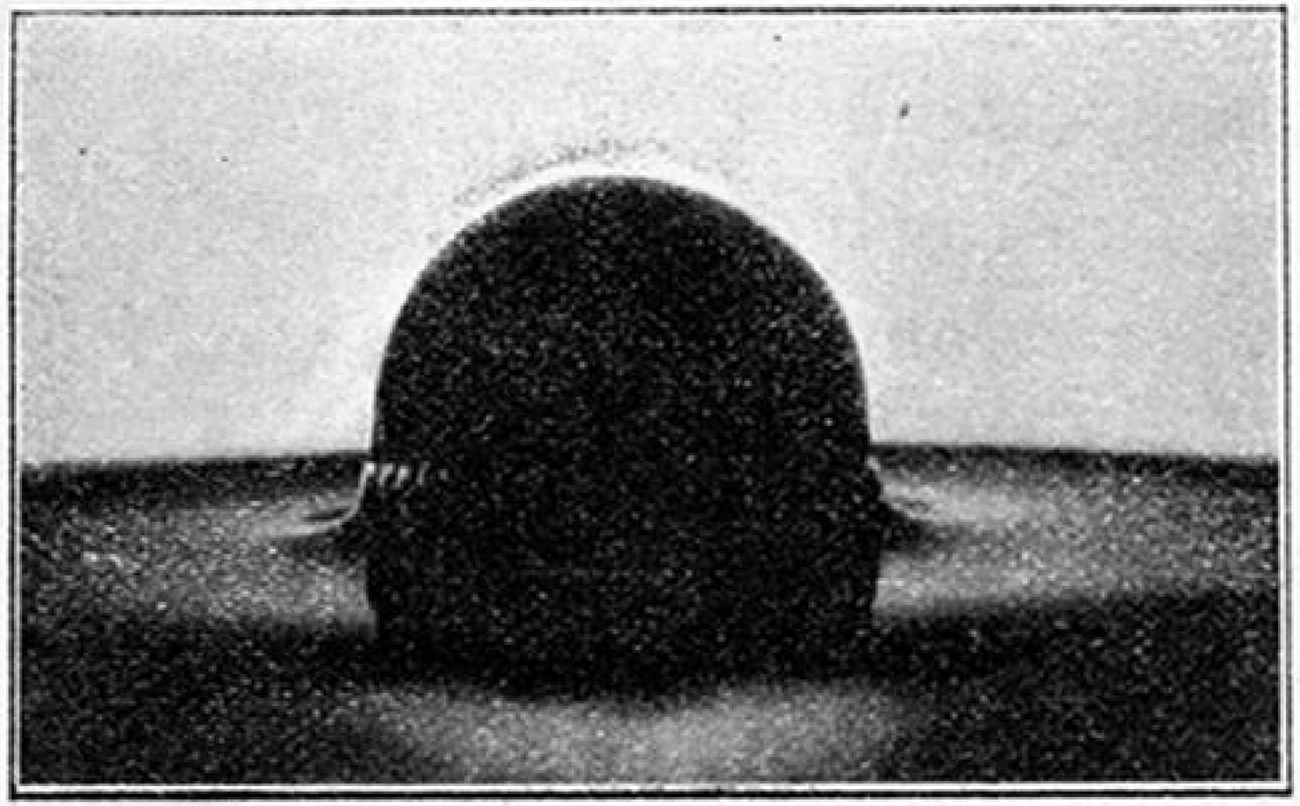


Fig. 3.

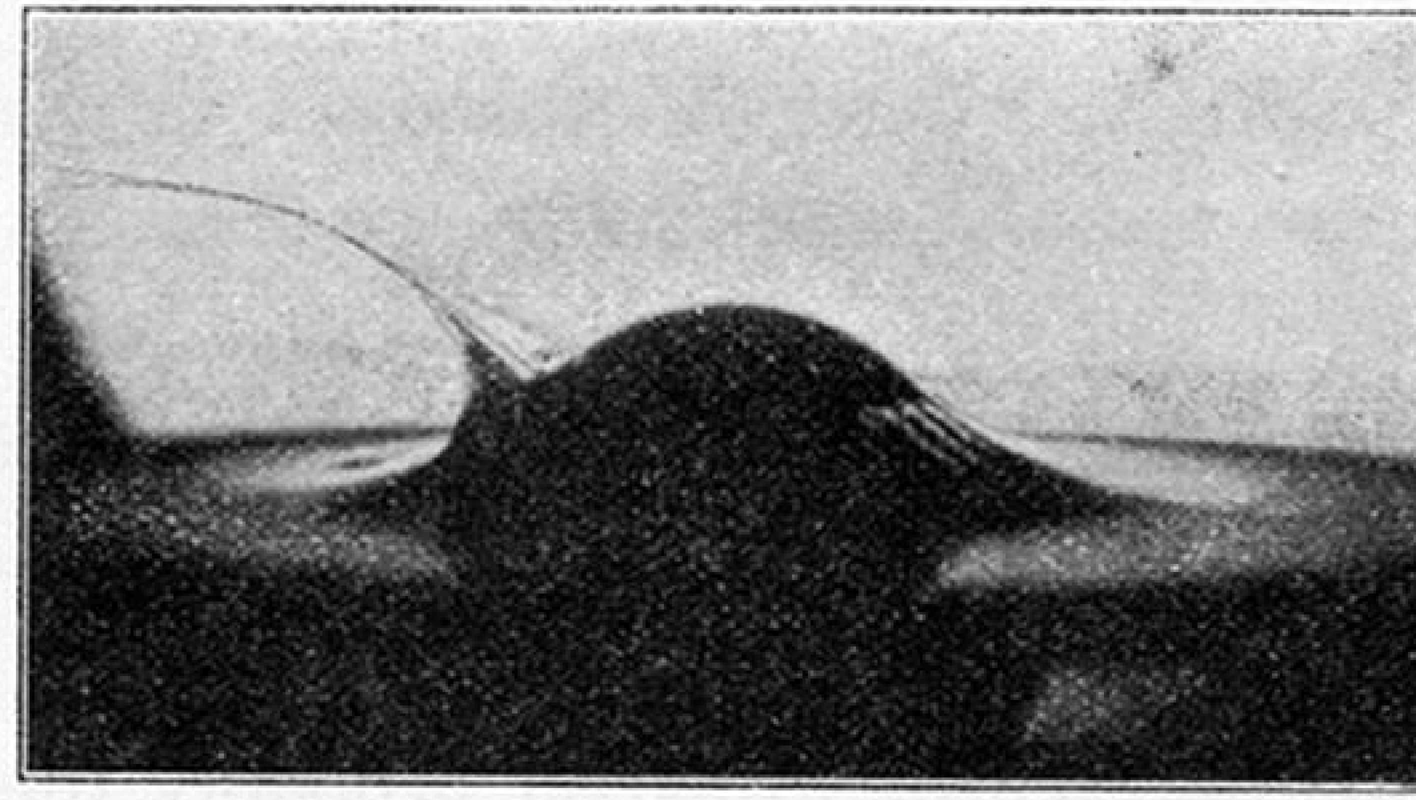
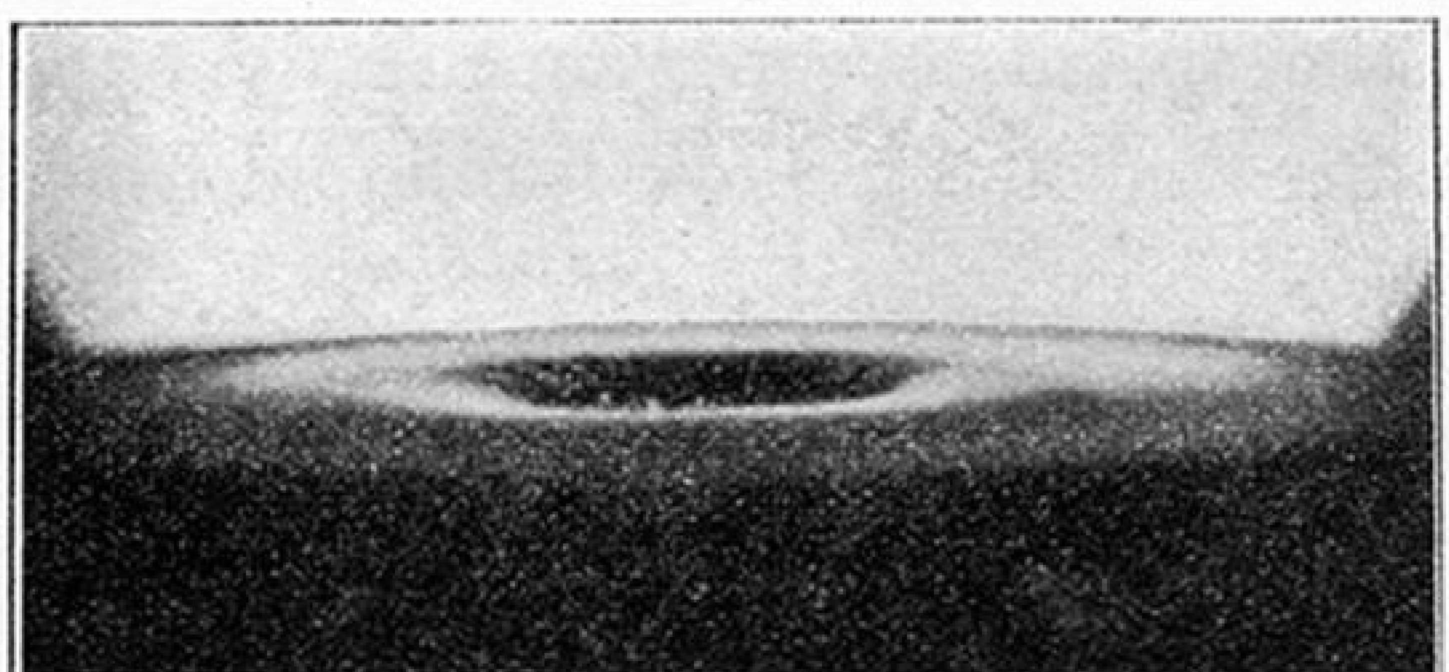


Fig. 4.



Series XXVII.

Fig. 1.

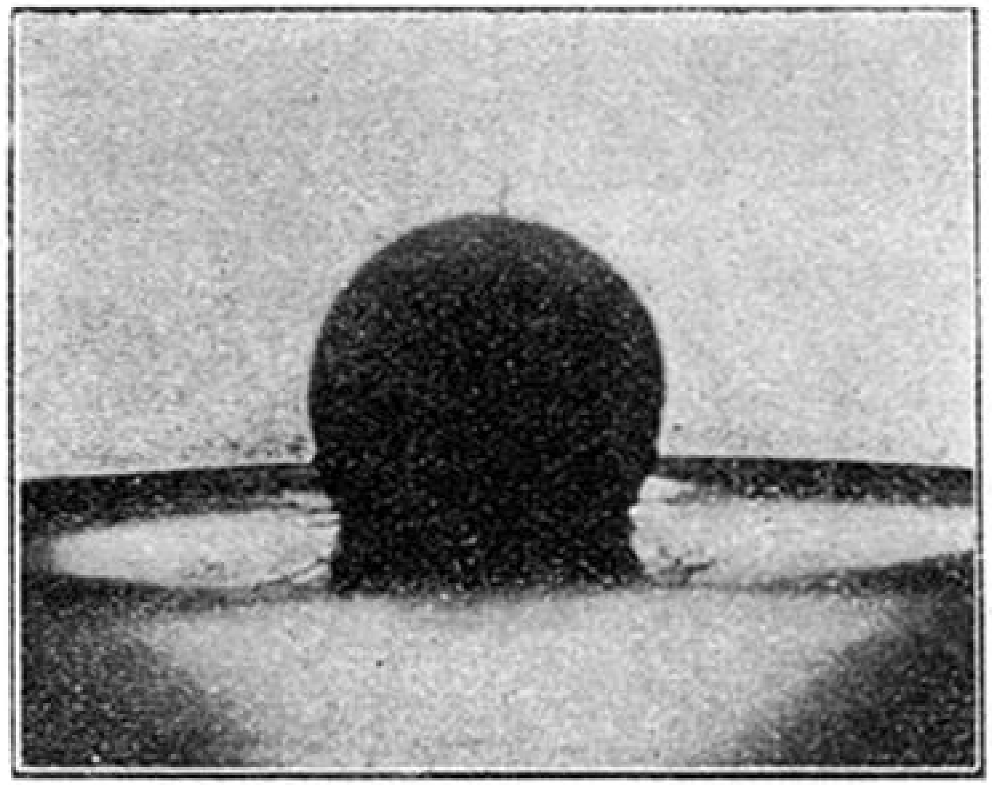


Fig. 2.

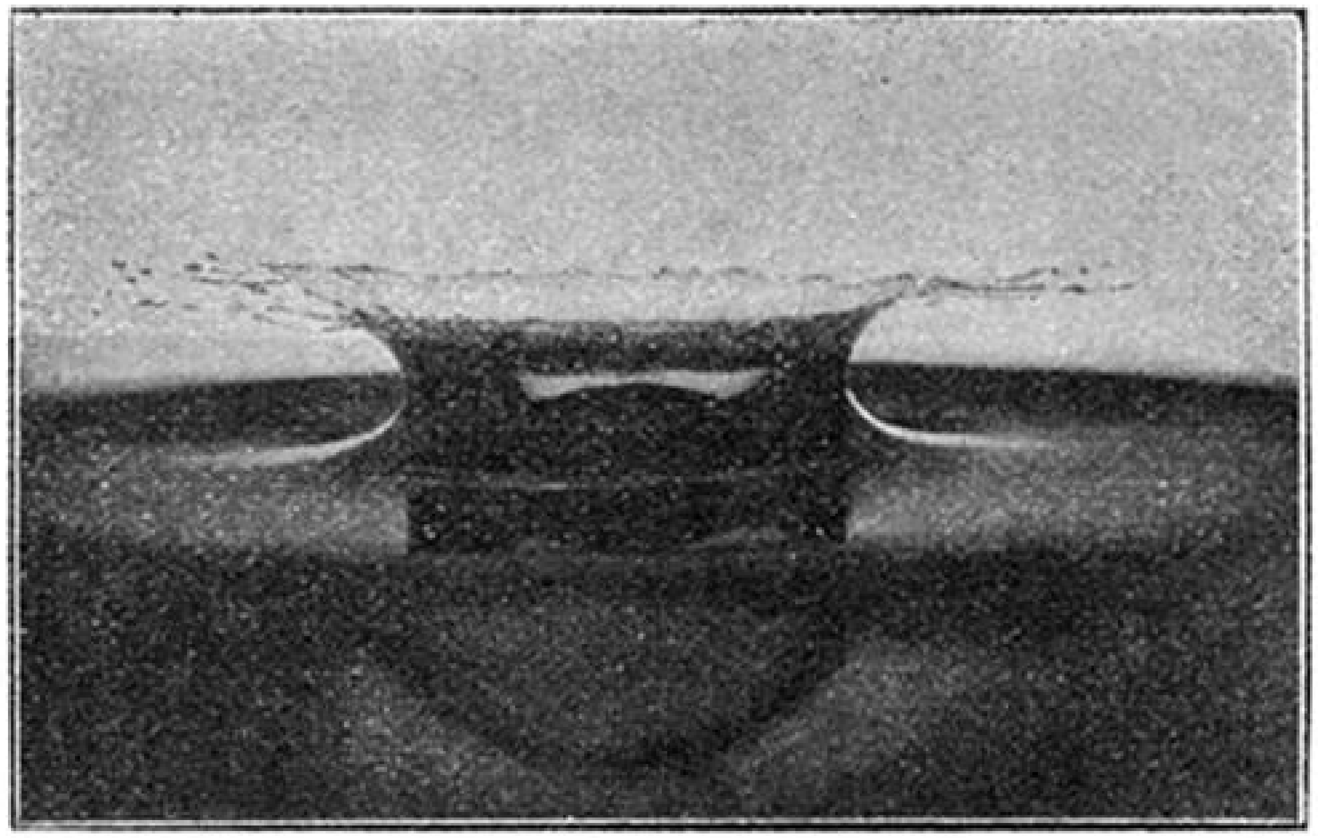
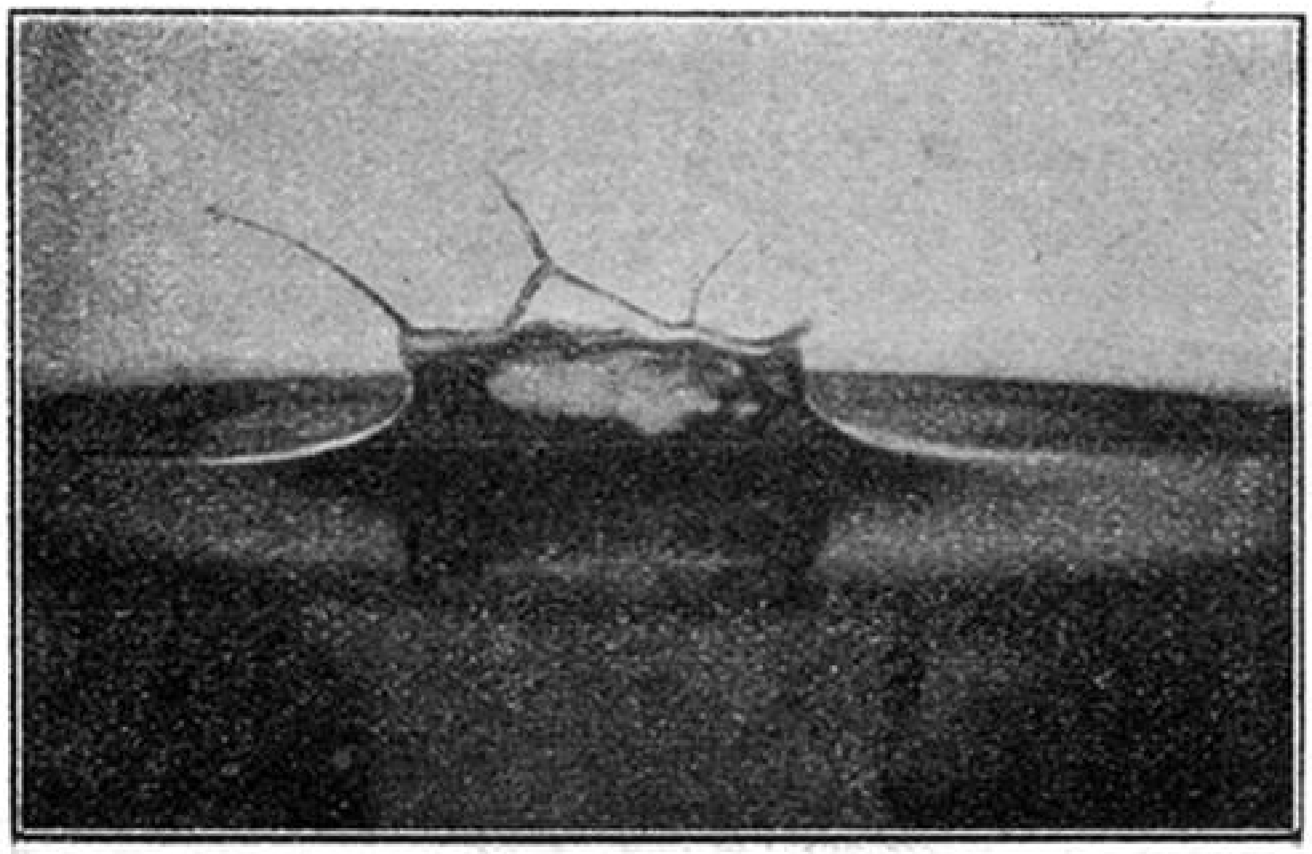
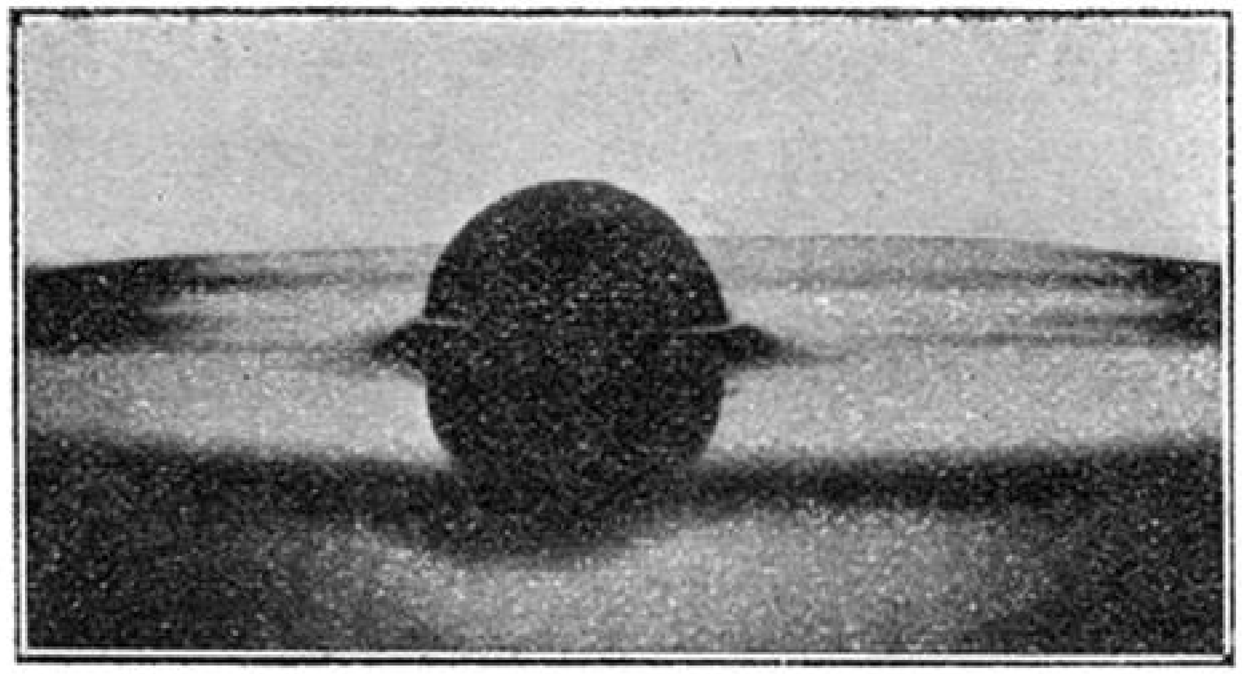


Fig. 3.



Series XXVIII.

Fig. 1.



Fig, 2.

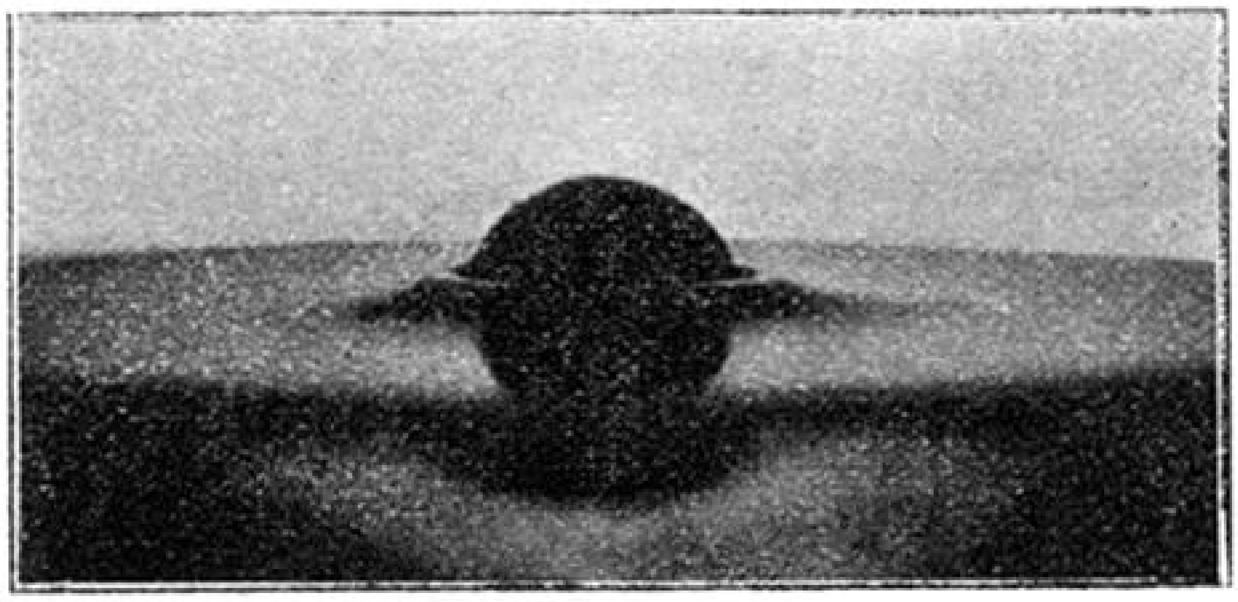


Fig. 3.

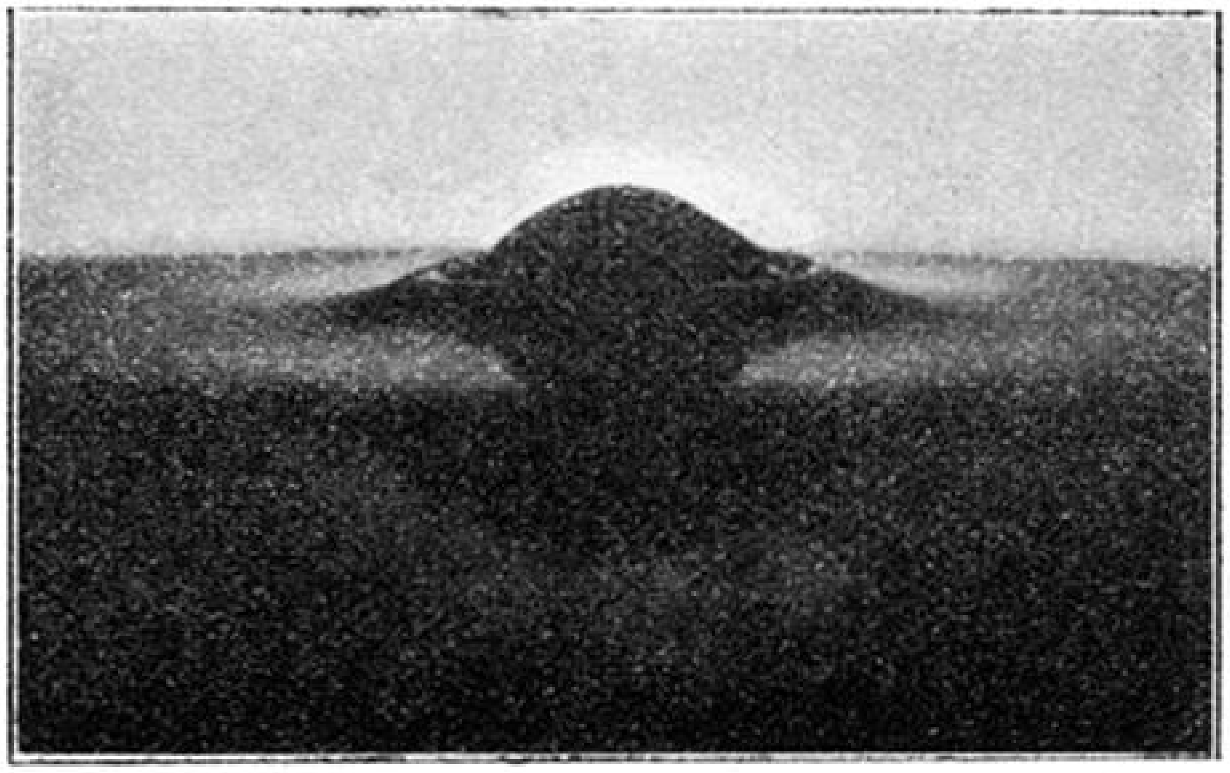


Fig. 4.

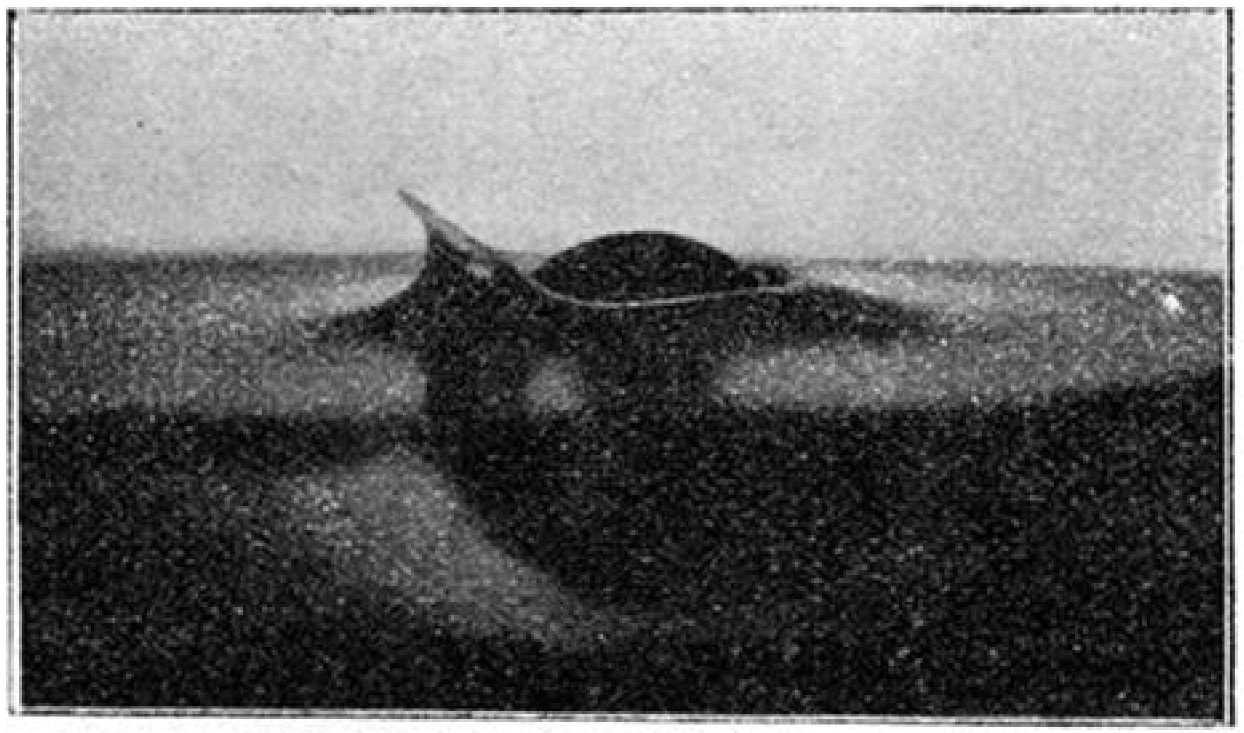


Fig. 5.

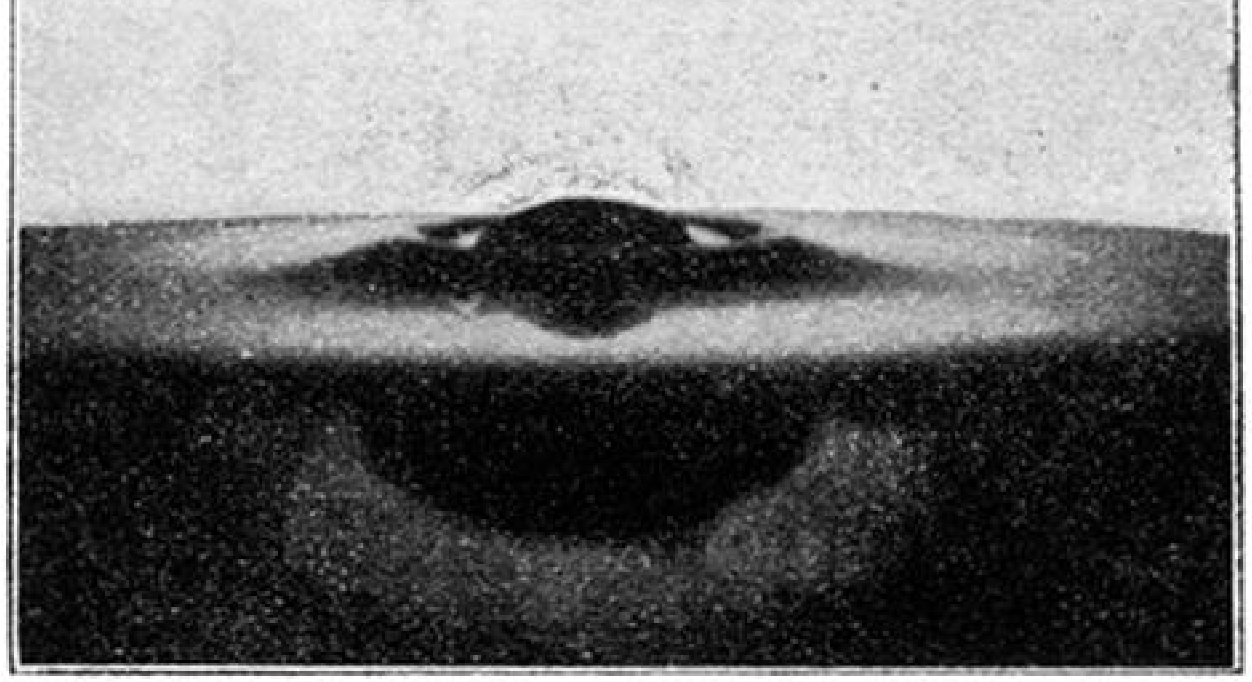
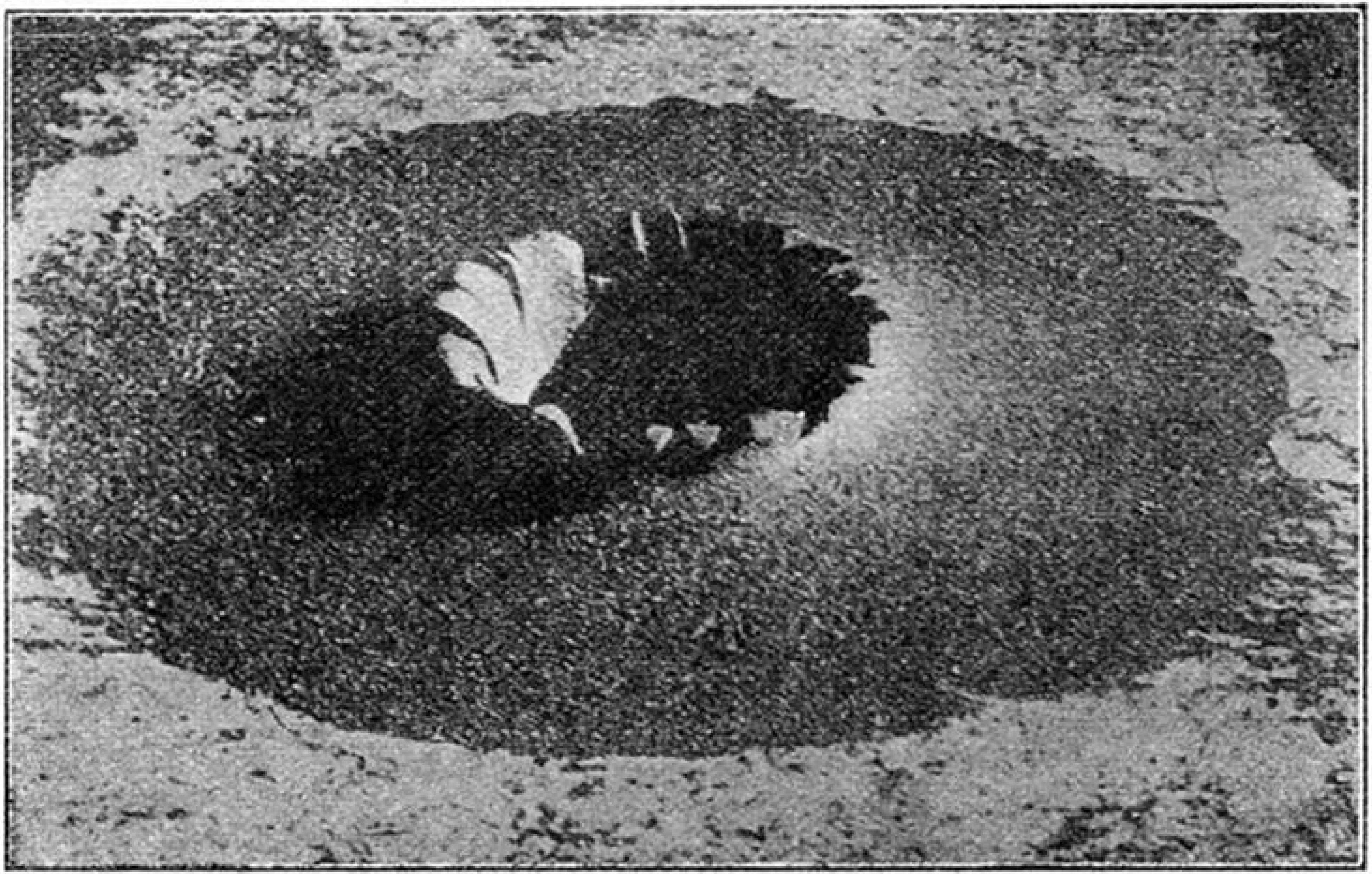


Fig. 5.



Series XII.

Fig. 1.

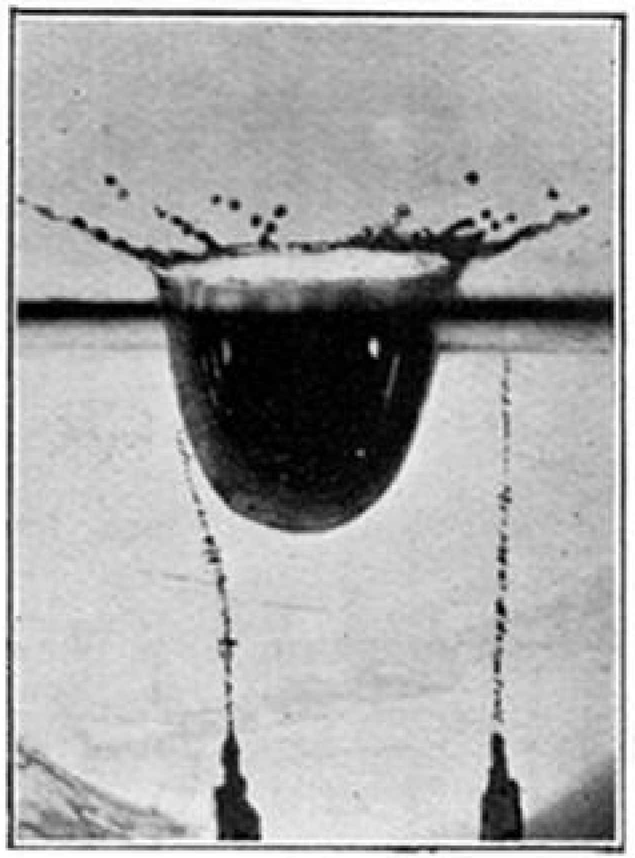


Fig. 2.

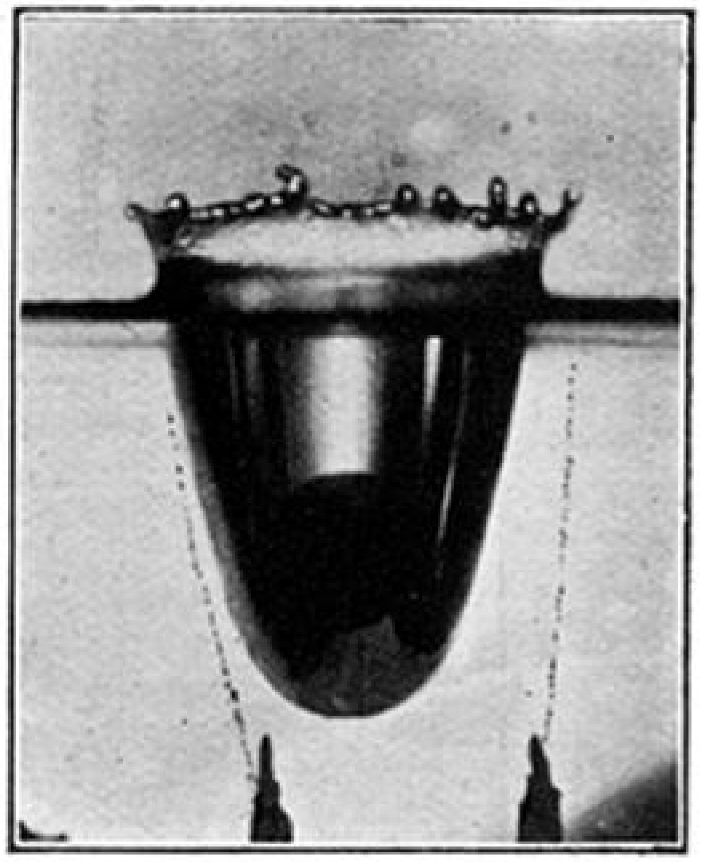


Fig. 3.

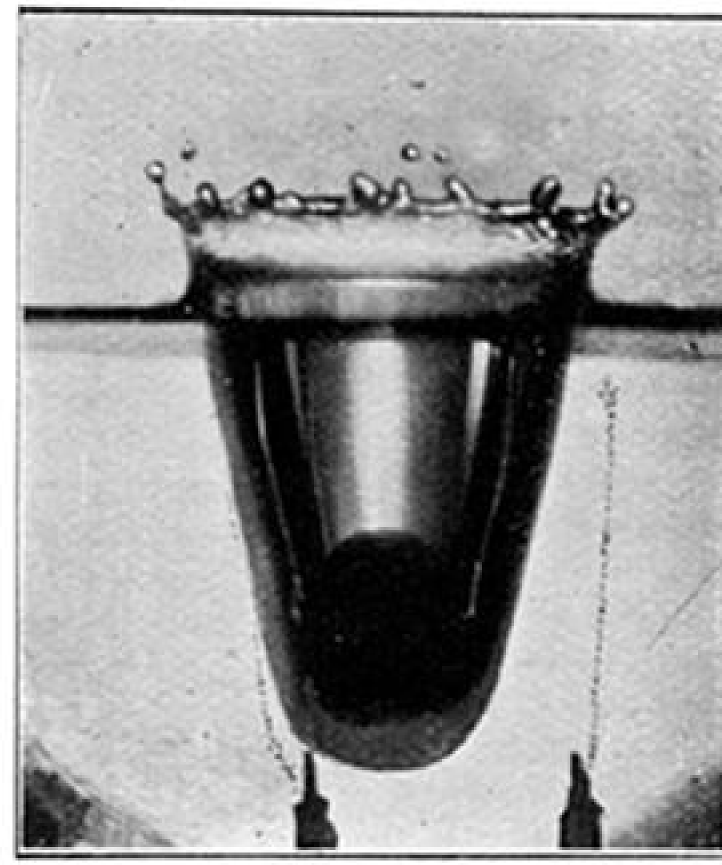


Fig. 5.

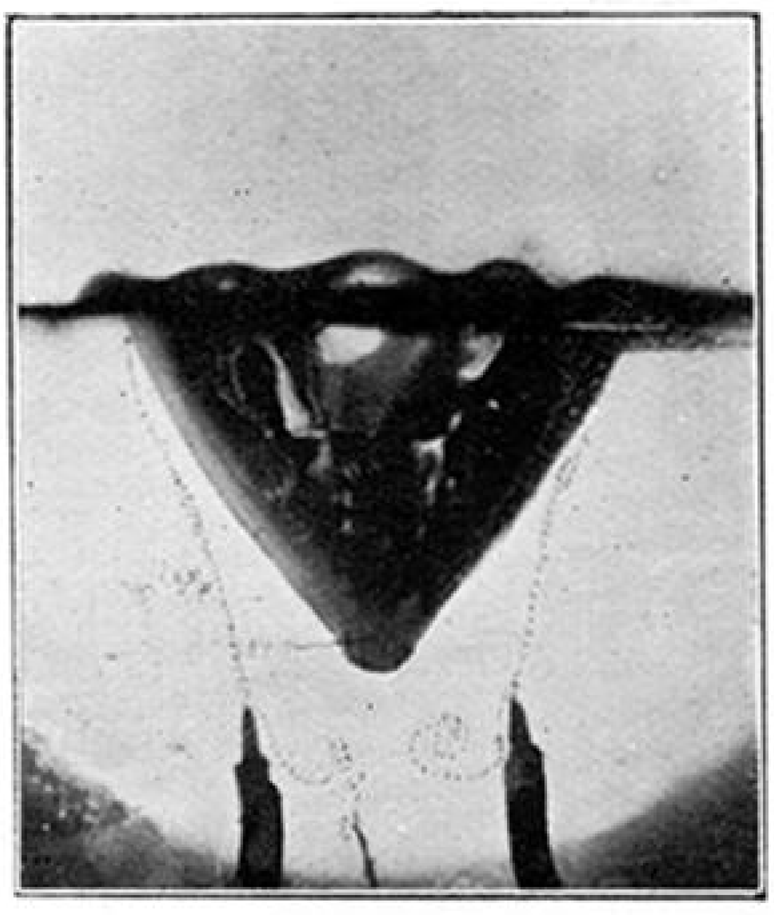
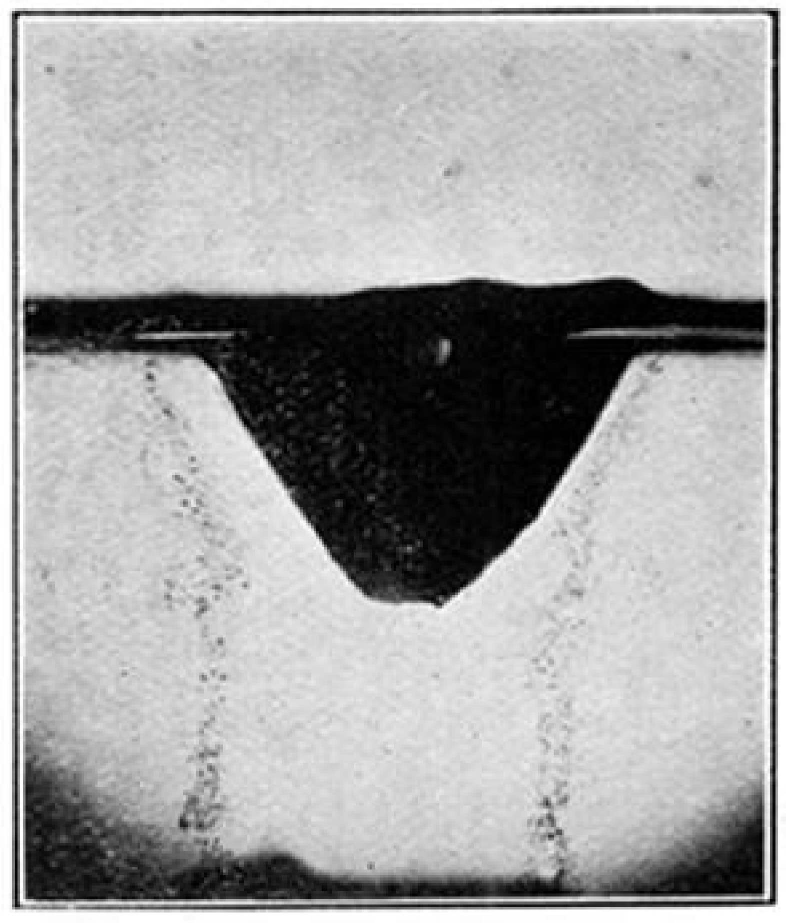
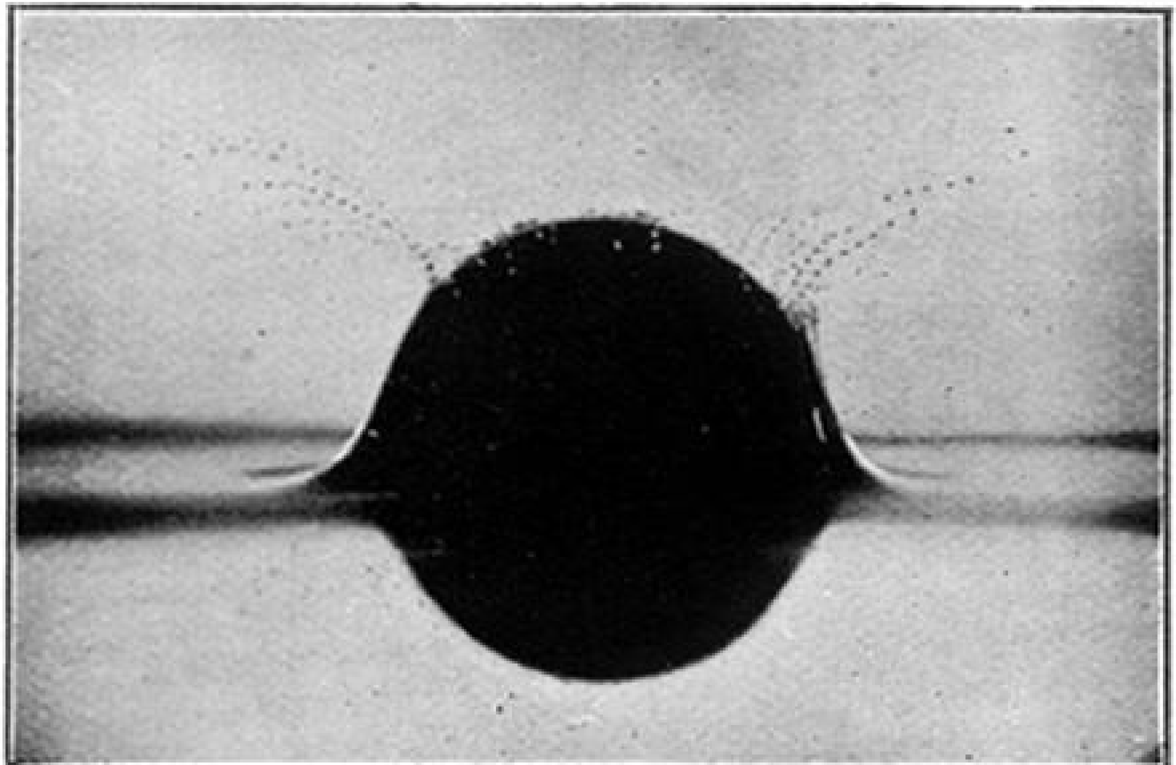


Fig. 6.



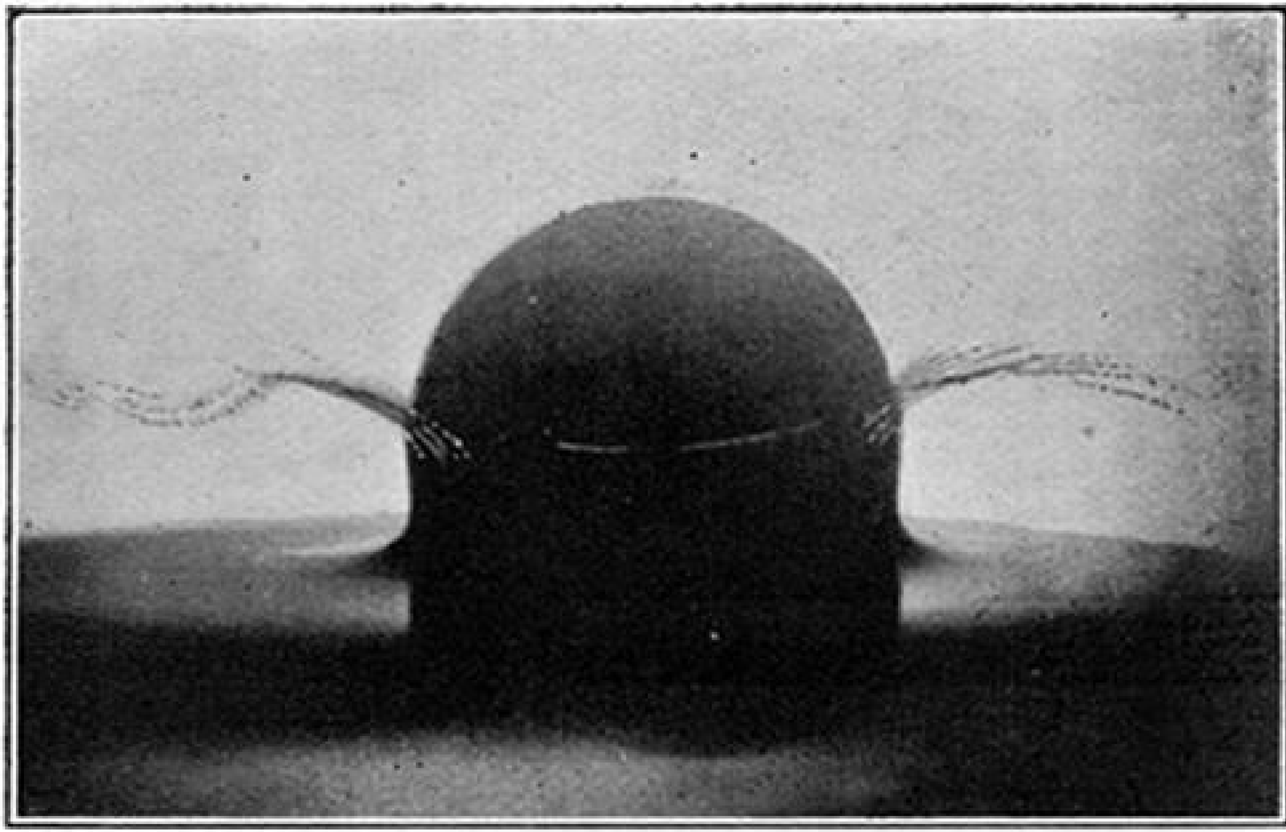
Series XIV.

Fig. 1.



Series XV.

Fig. 2.



Series XVII.

Fig. 1.

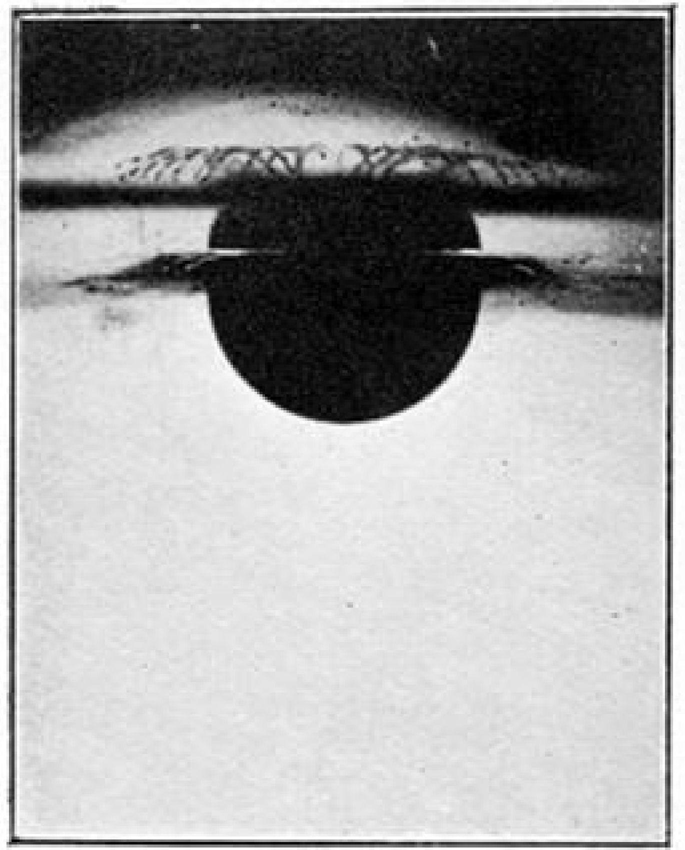


Fig. 2.

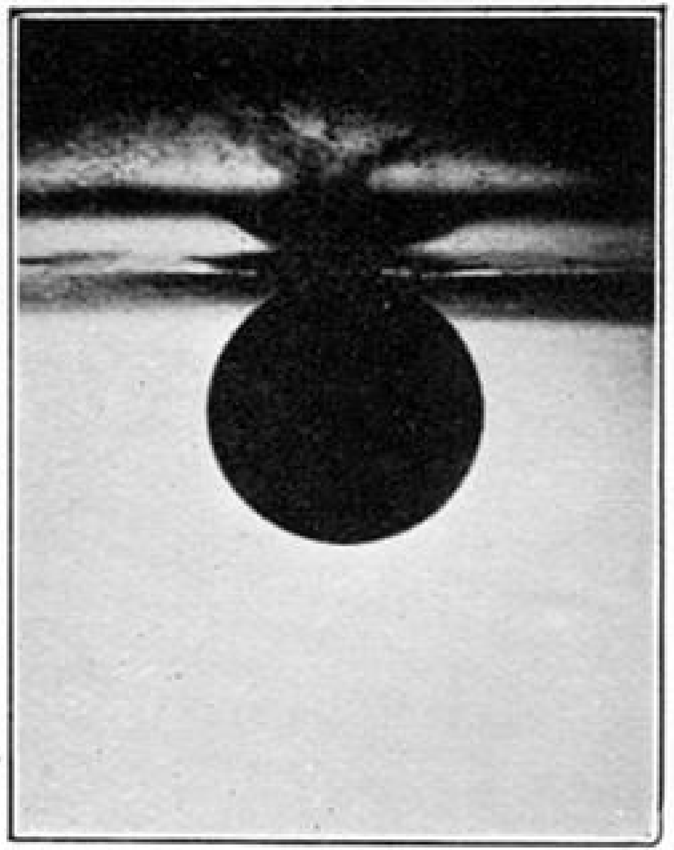
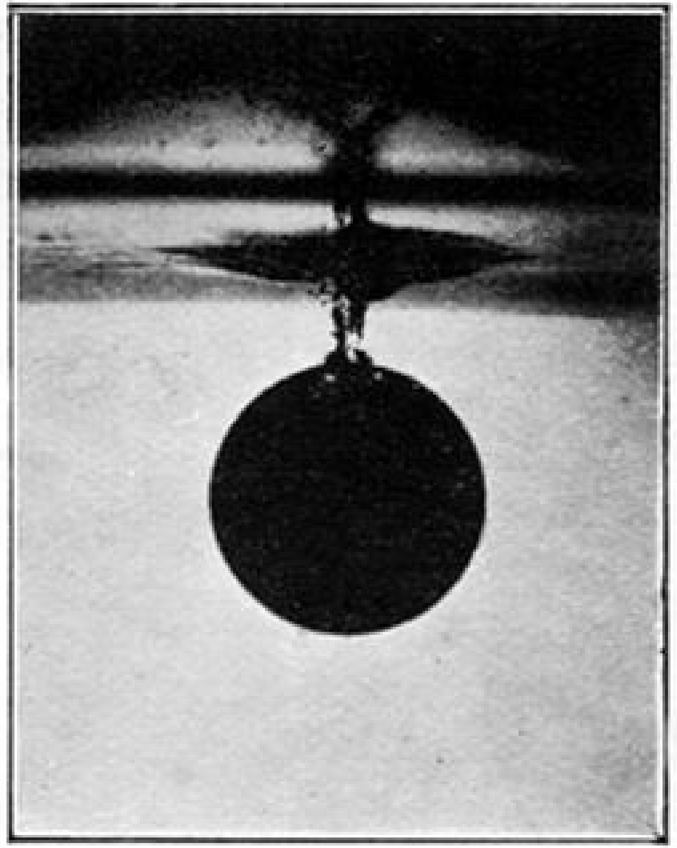


Fig. 3.



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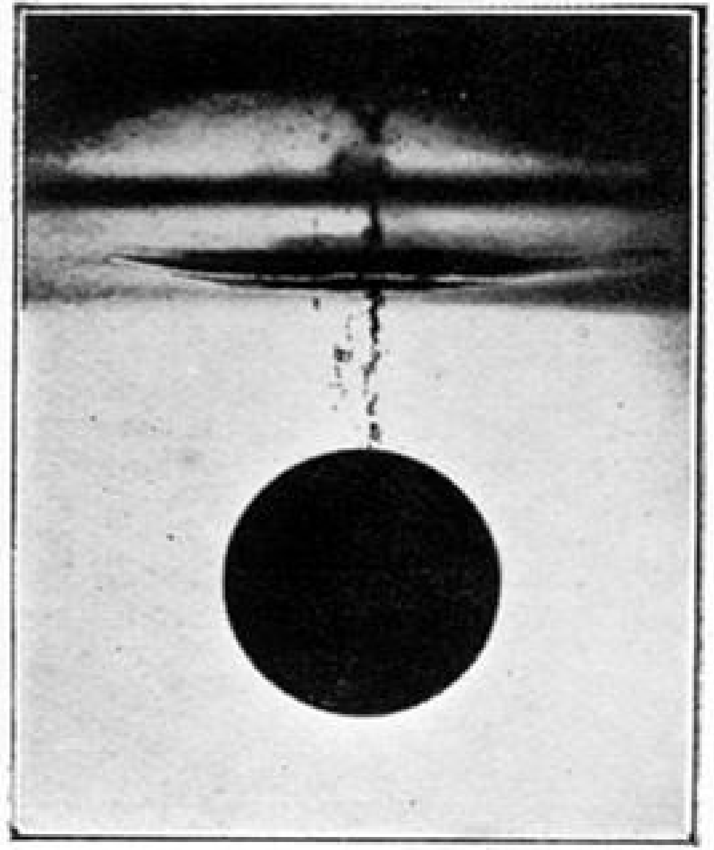


Series XVIII.

Fig. 1.

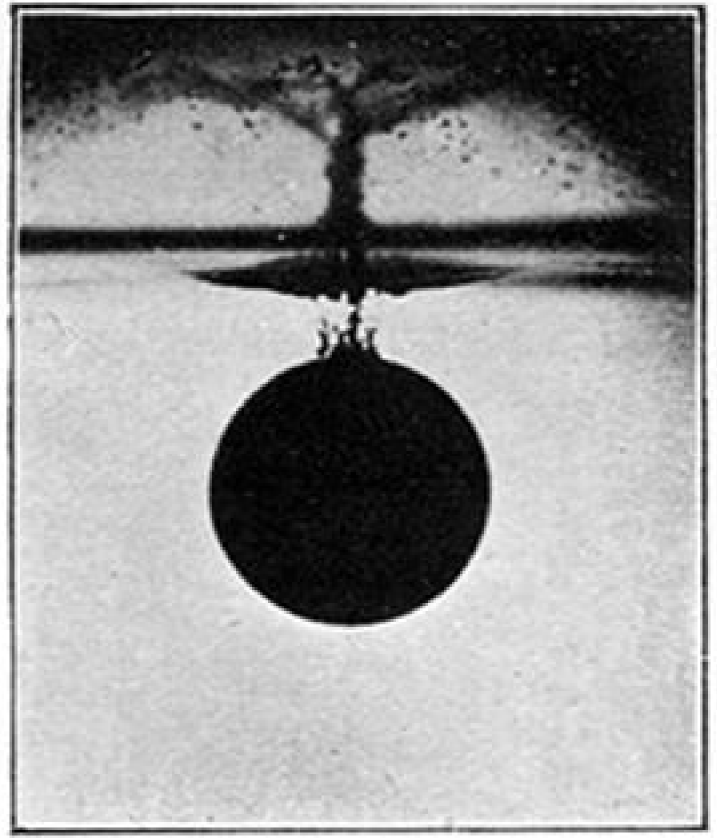


Fig. 2.

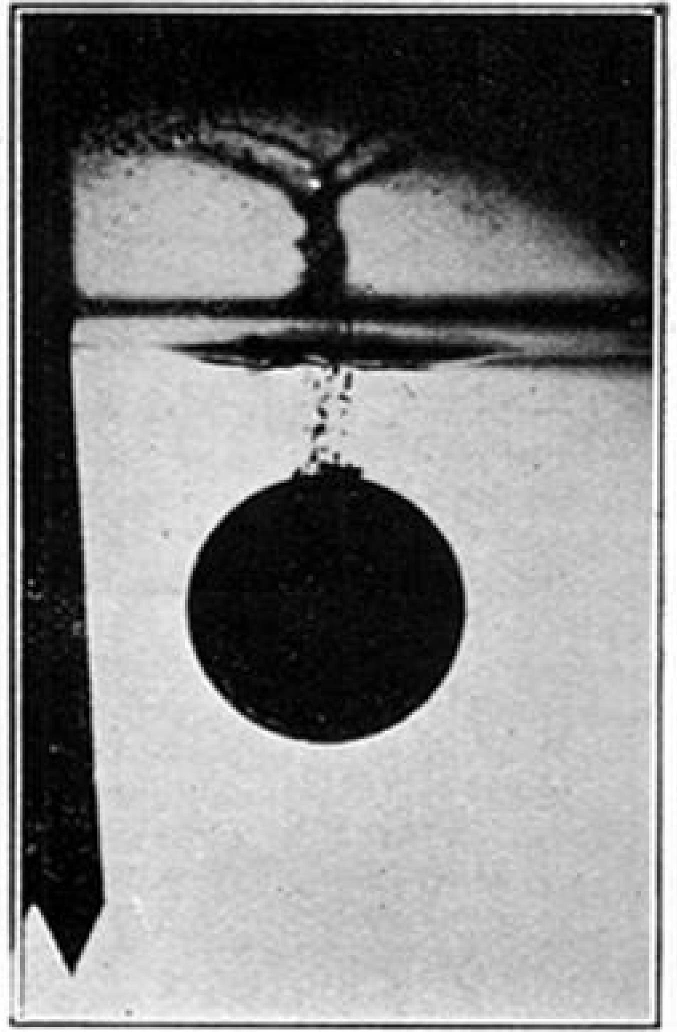


Fig. 3.

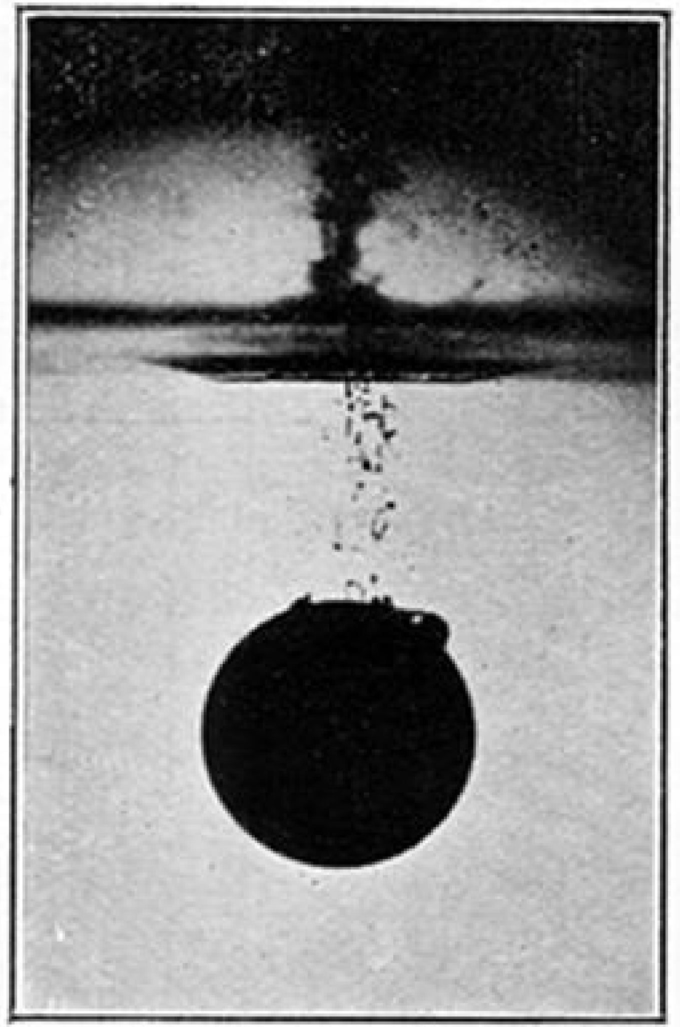
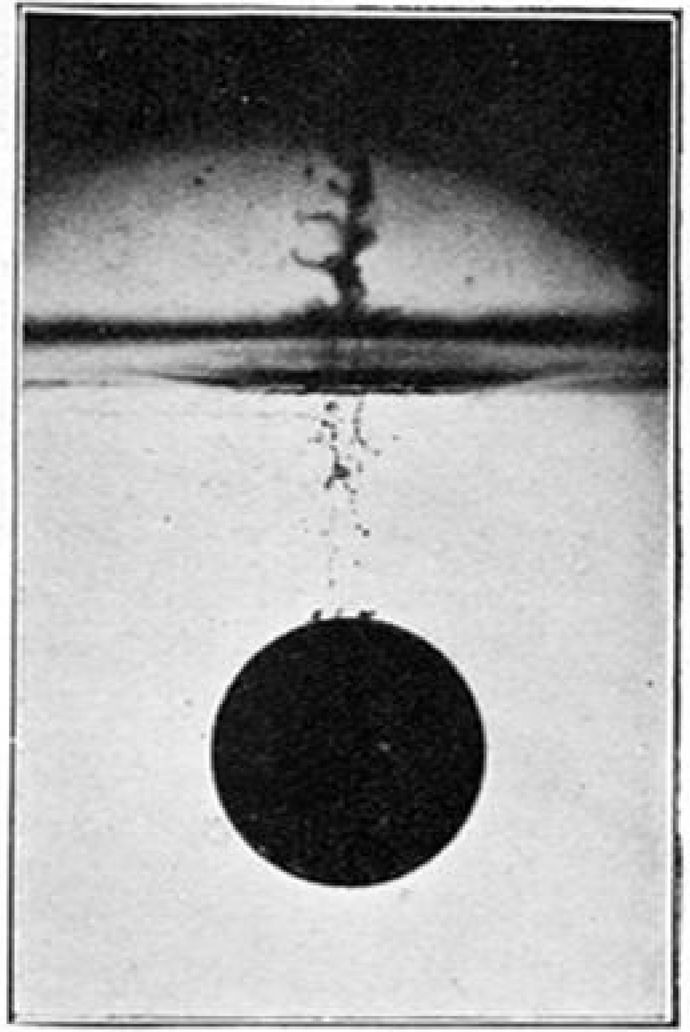


Fig. 4.



Series XIX.

Fig. 1.

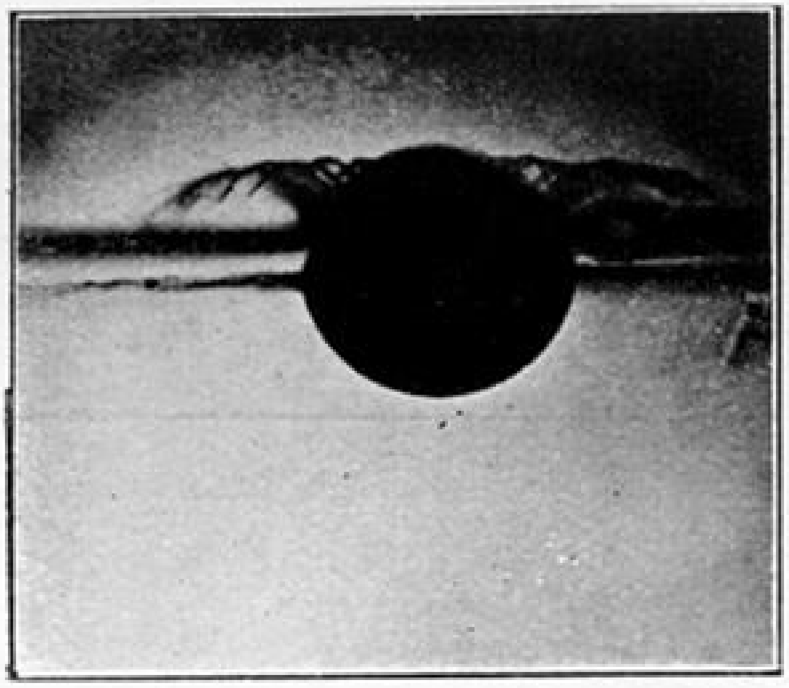


Fig. 2.

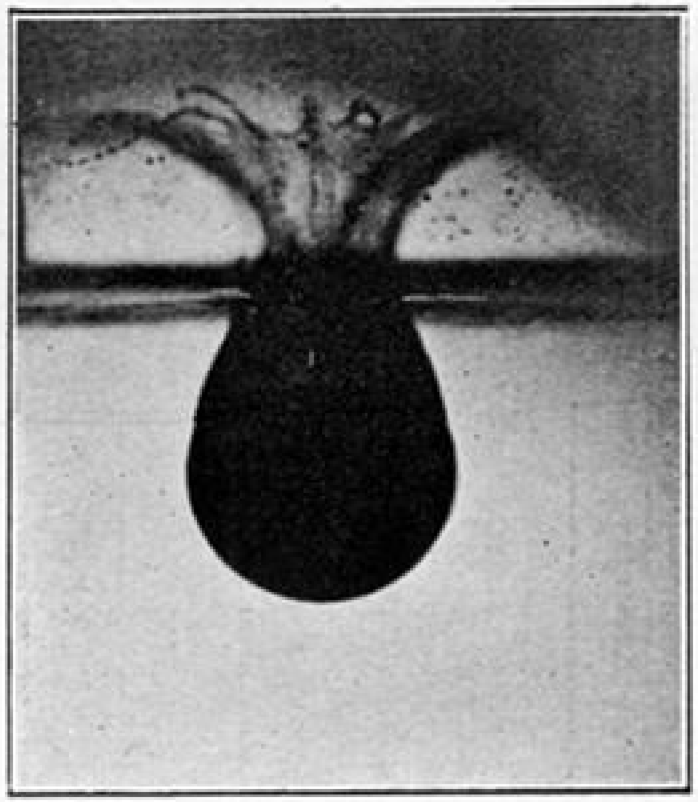


Fig. 3.

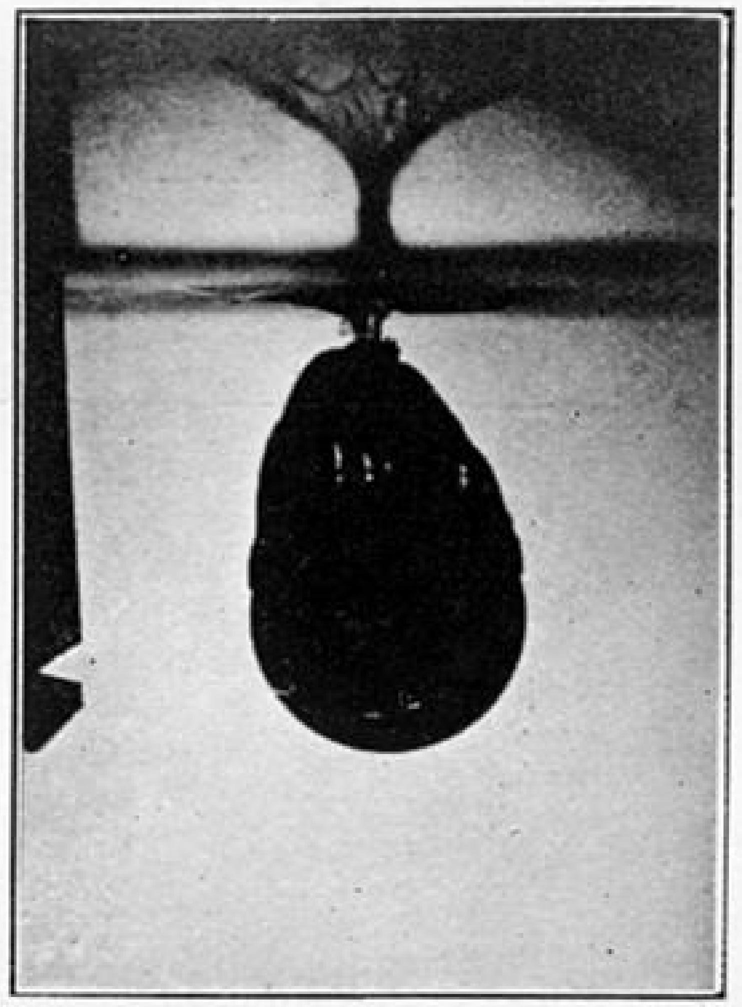


Fig. 4.

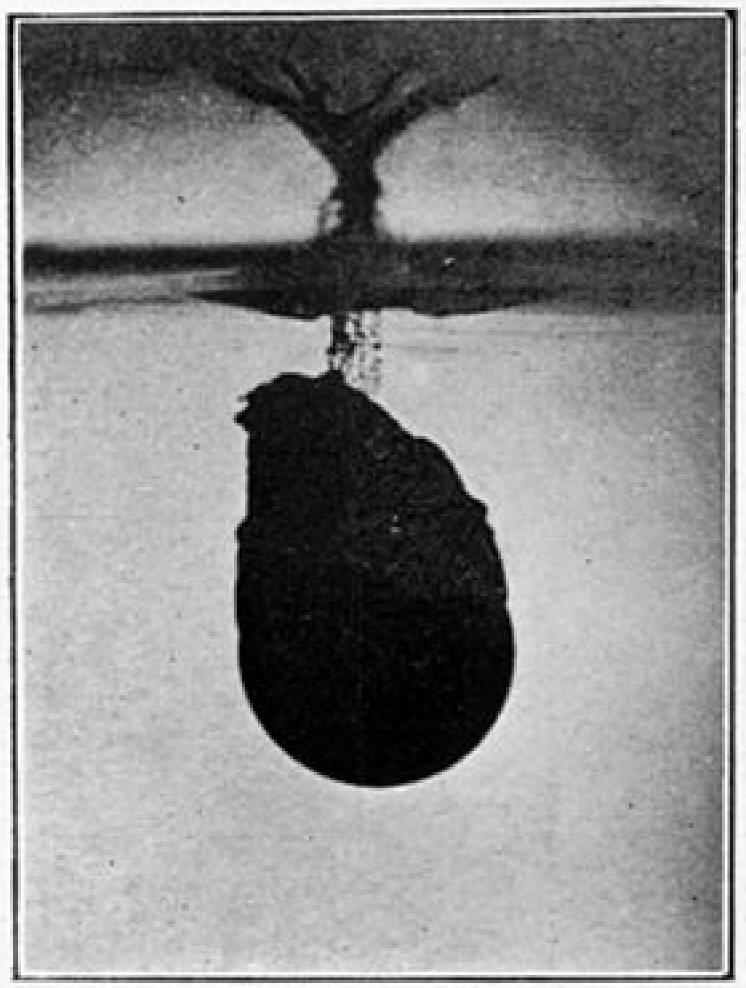


Fig. 5.

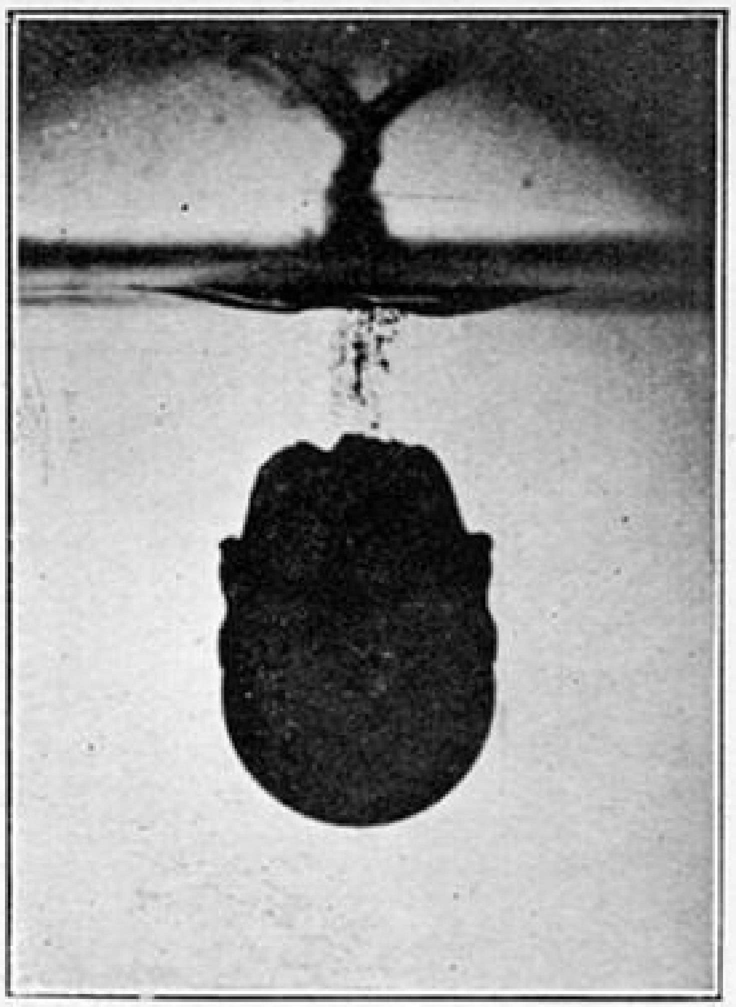
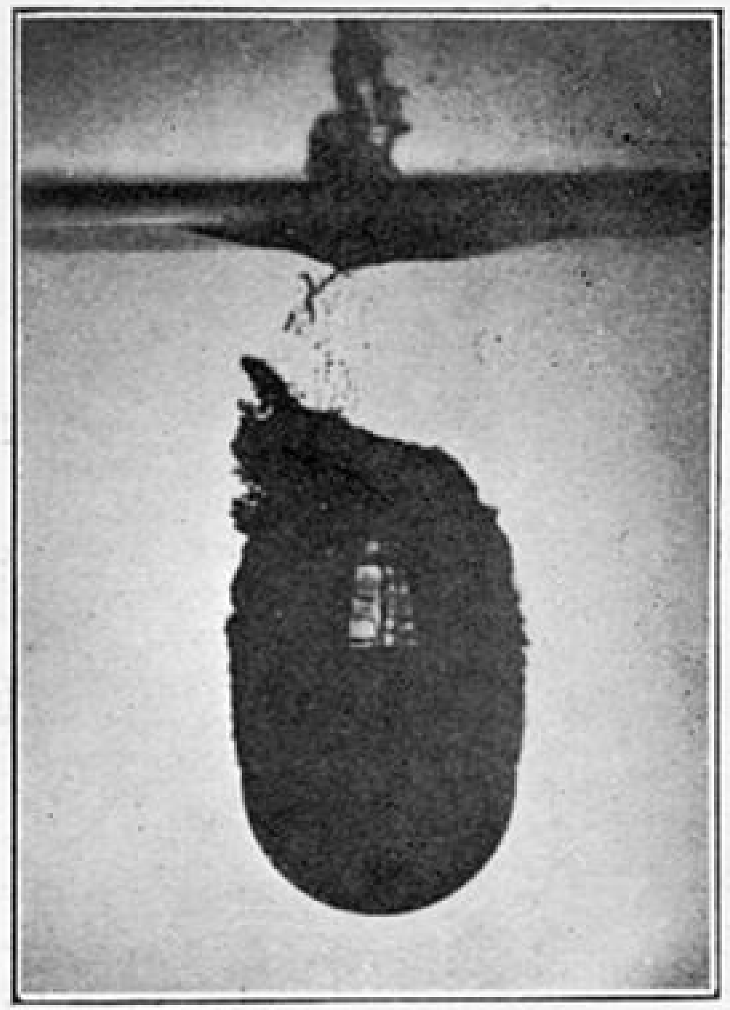


Fig. 6.



Series XX.

Fig. 3.

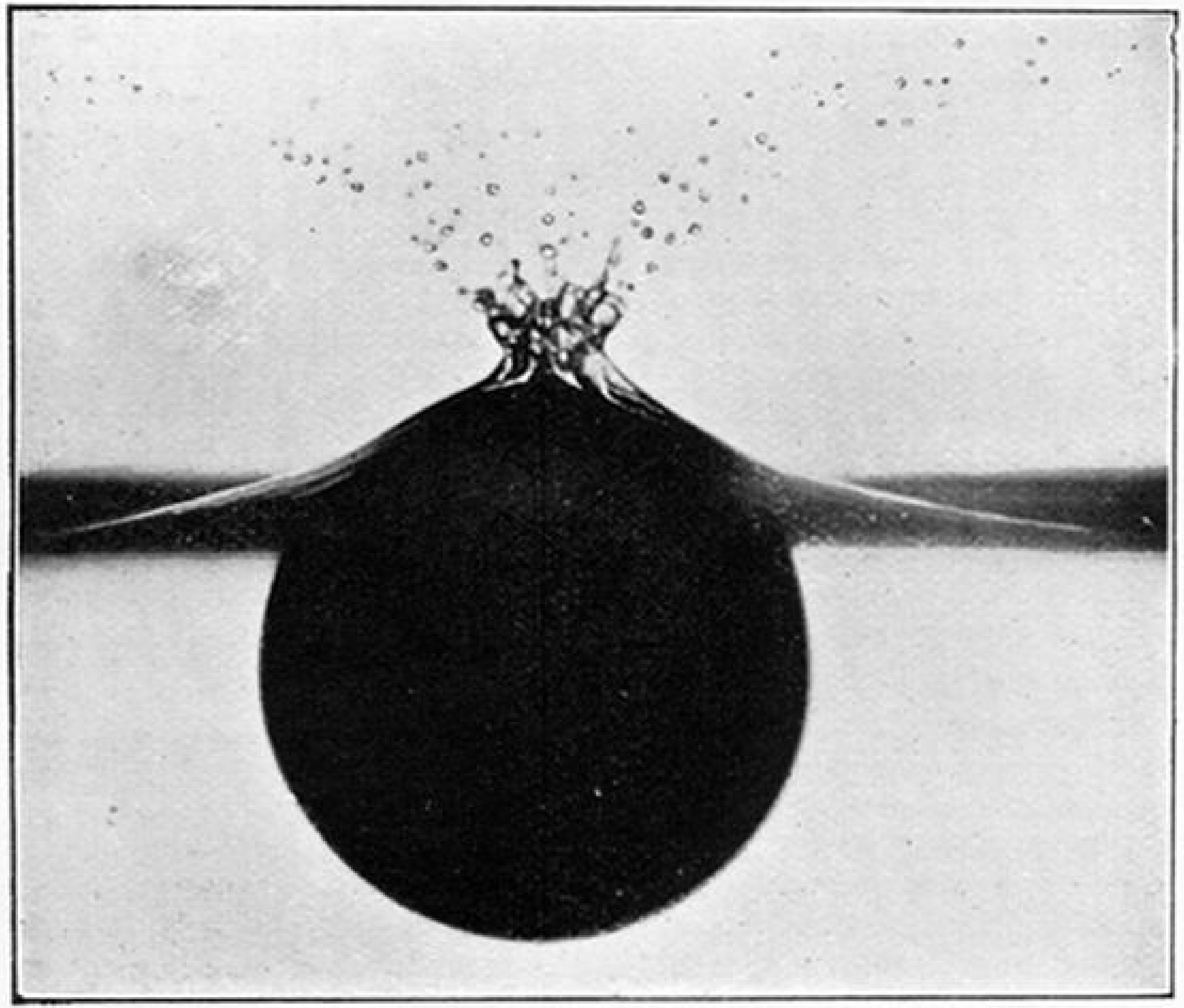
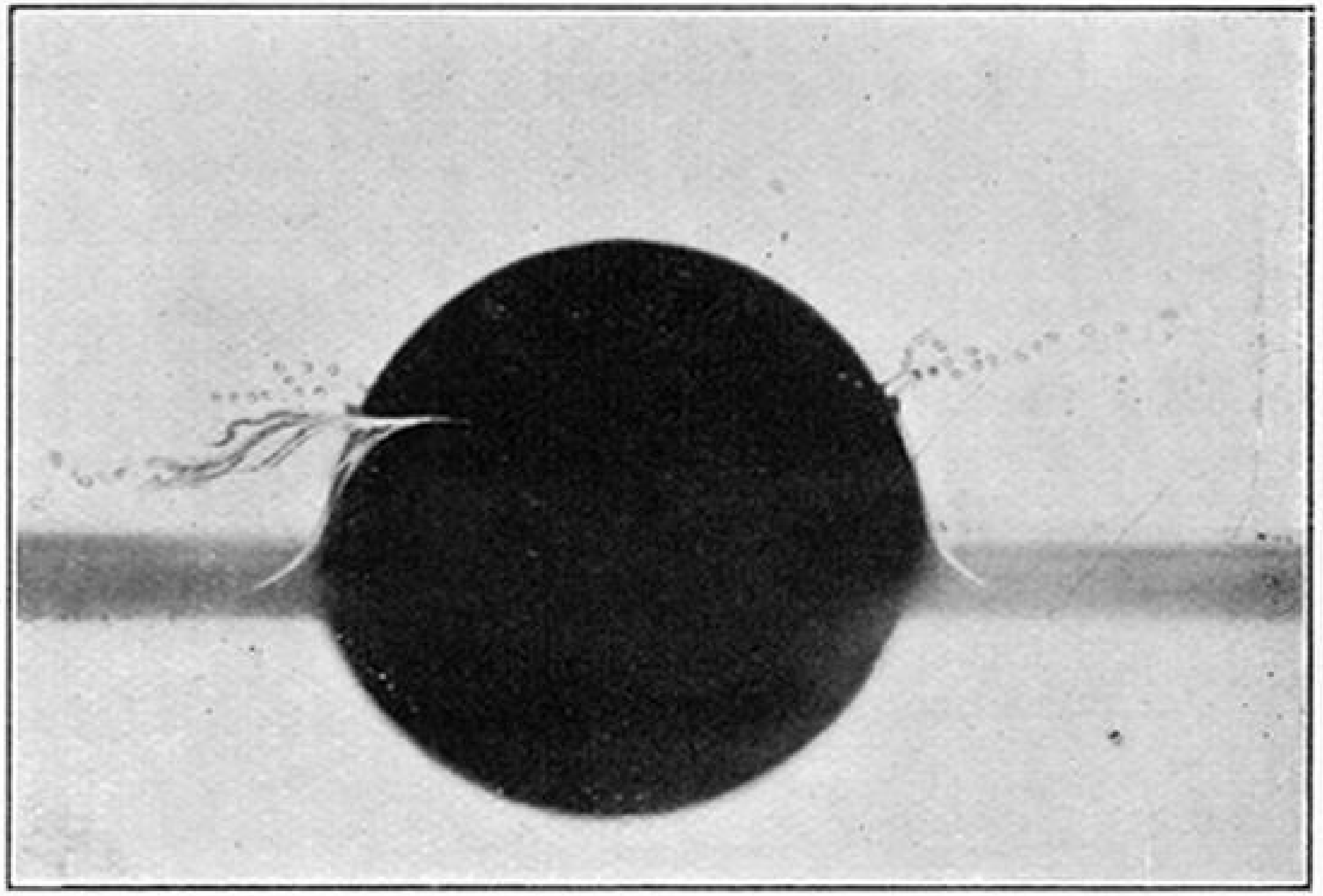


Fig. 4.



Series XXVI.

Fig. 1.

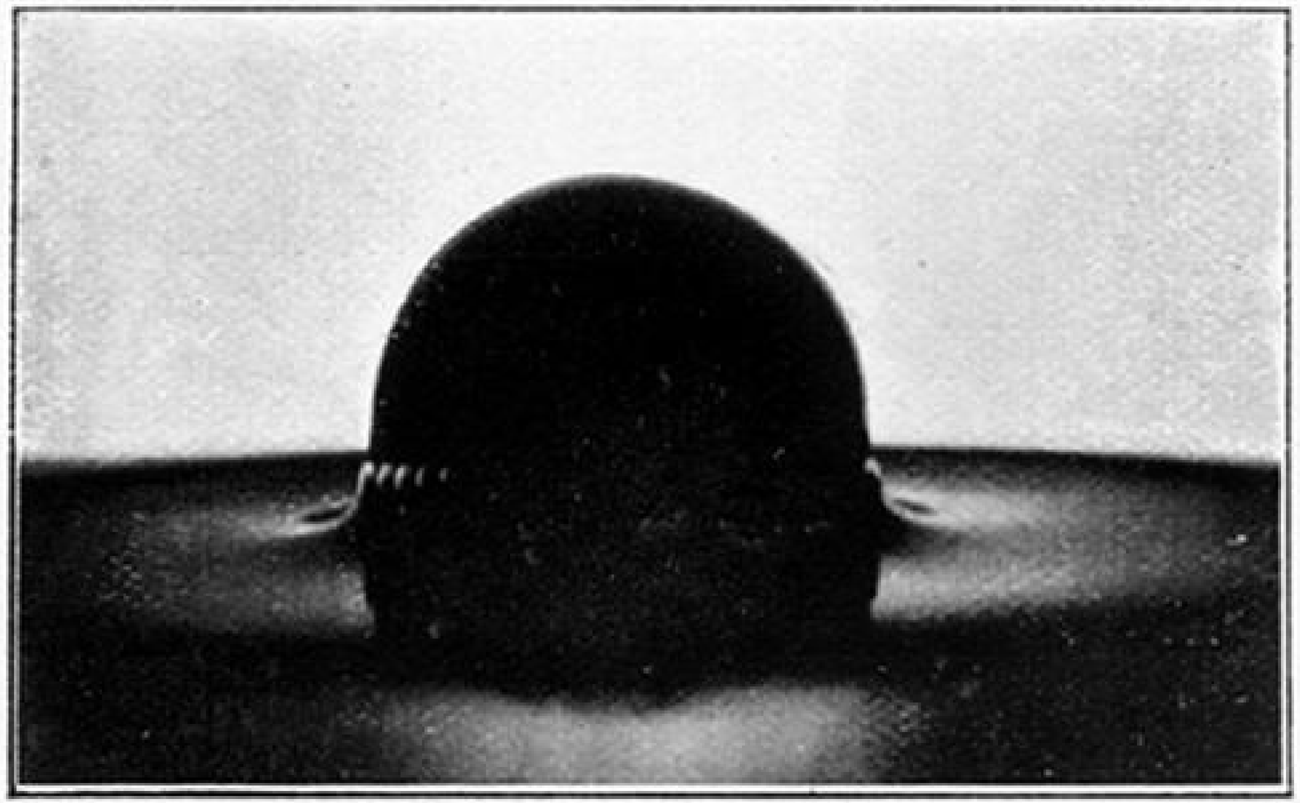


Fig. 3.

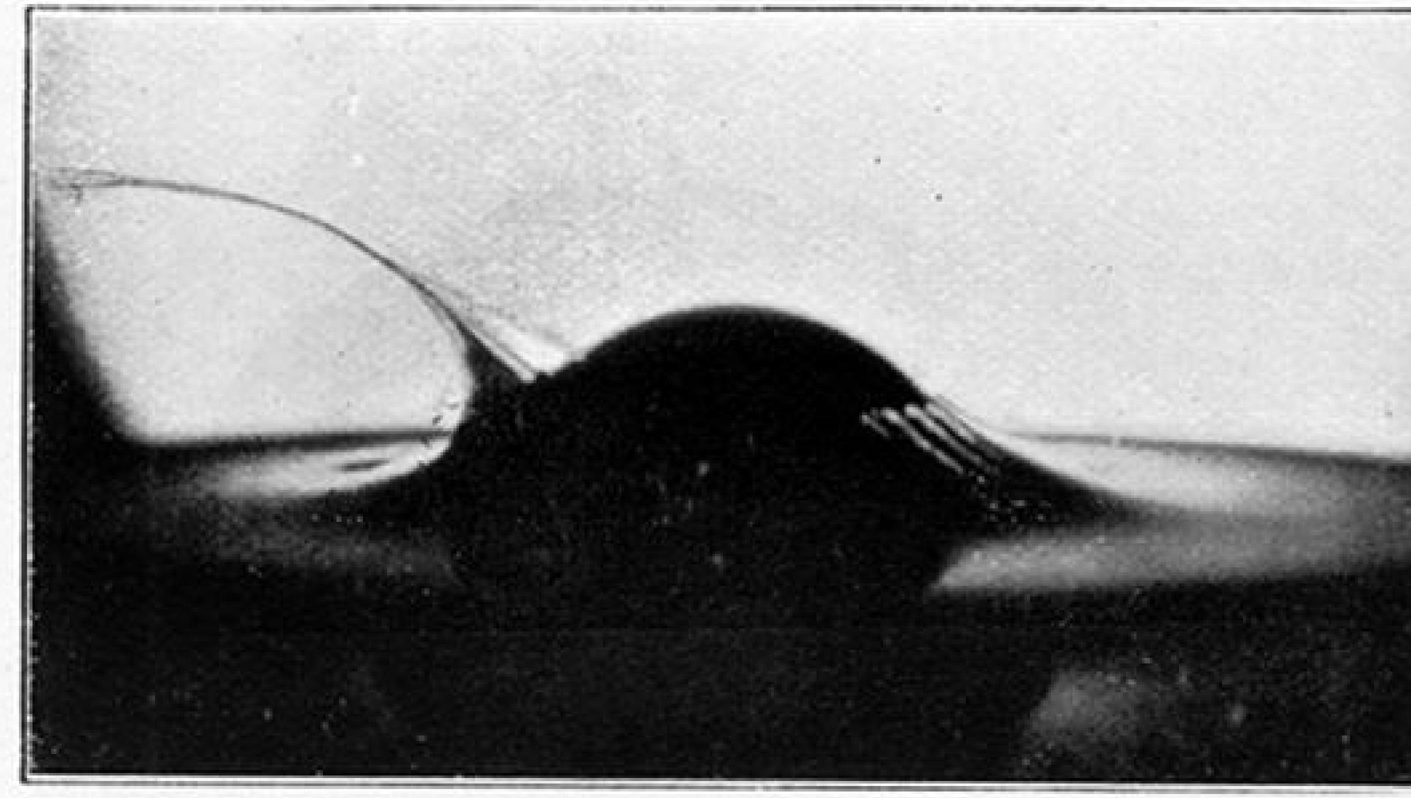
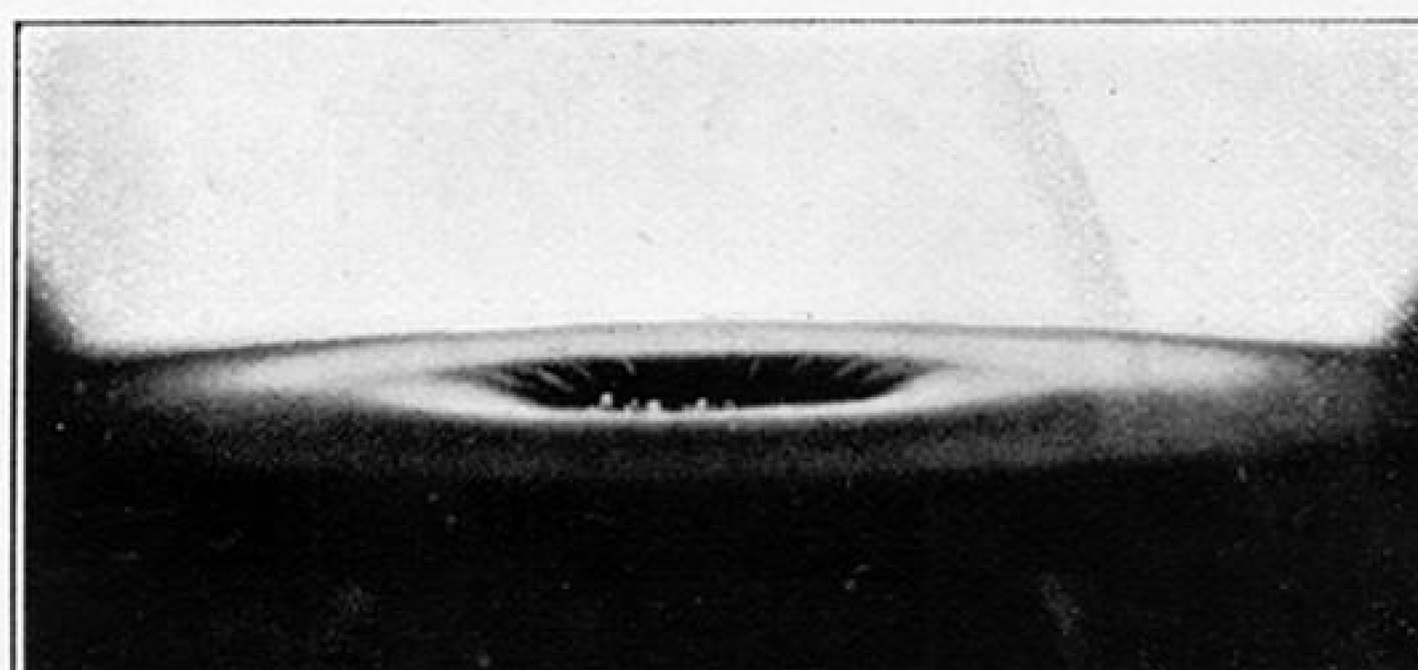


Fig. 4.



Series XXVII.

Fig. 1.

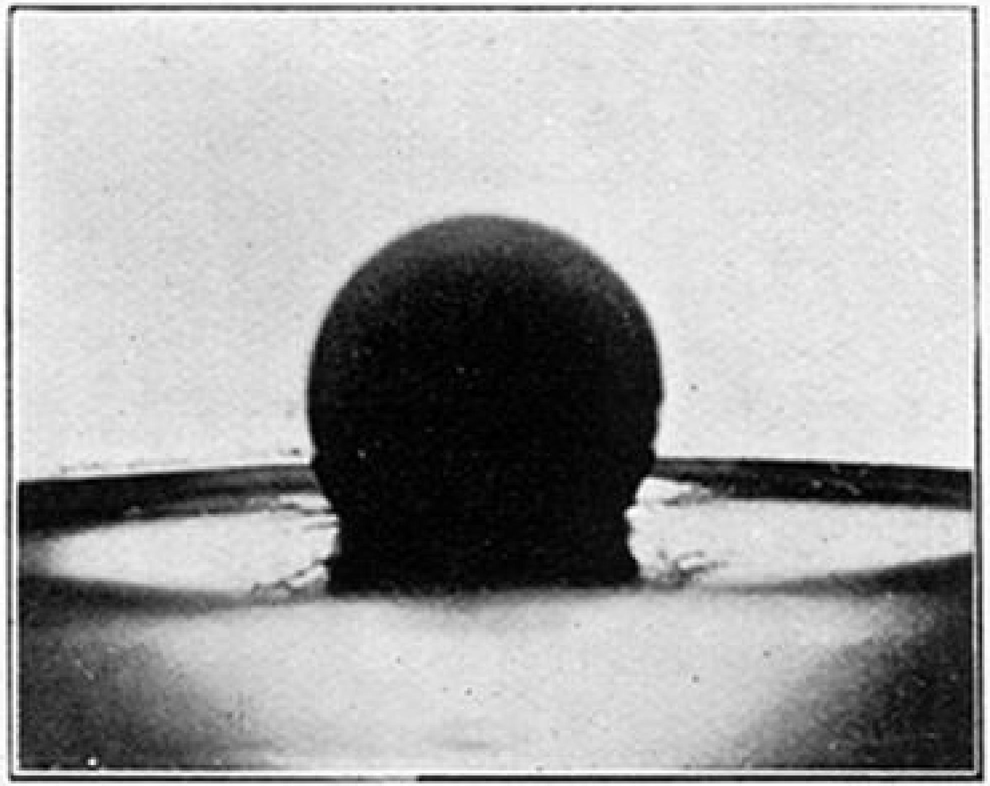
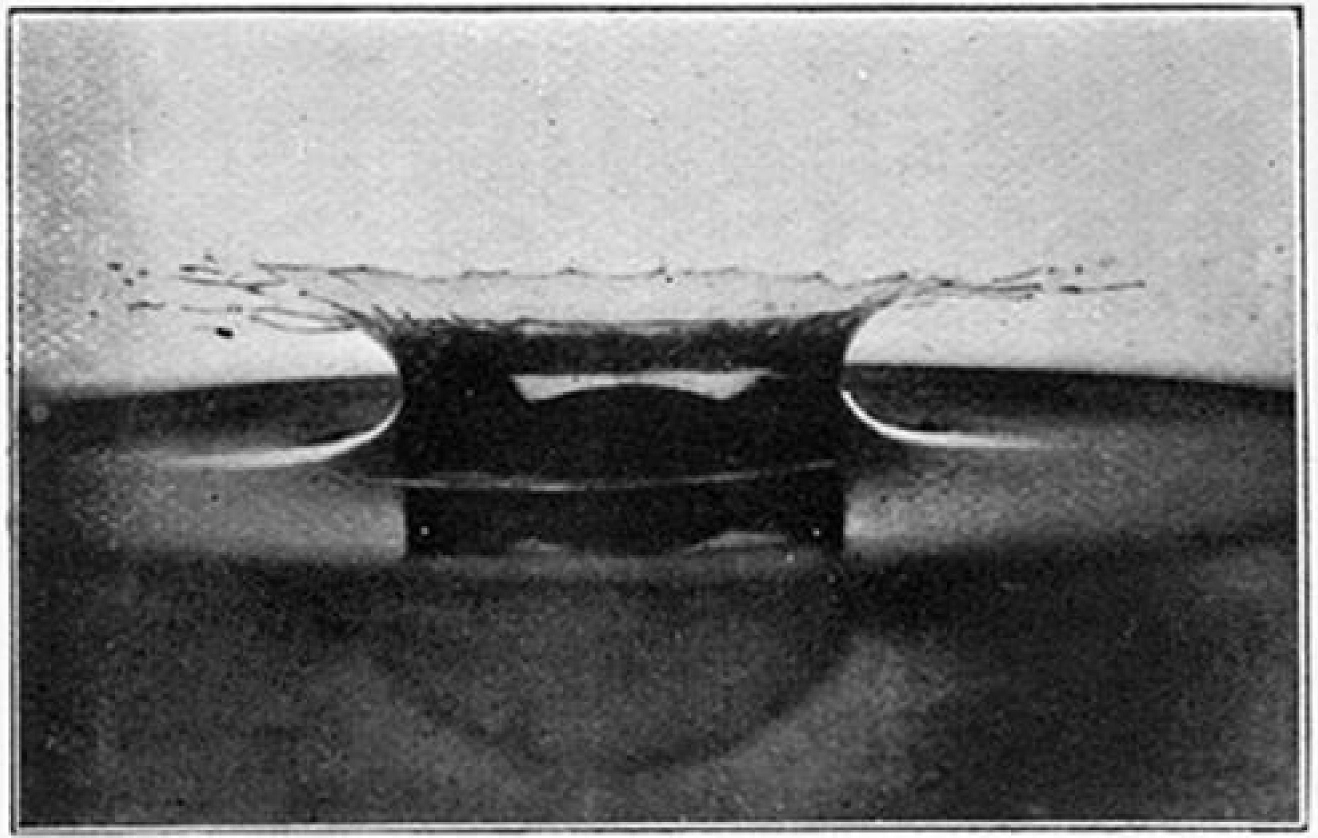
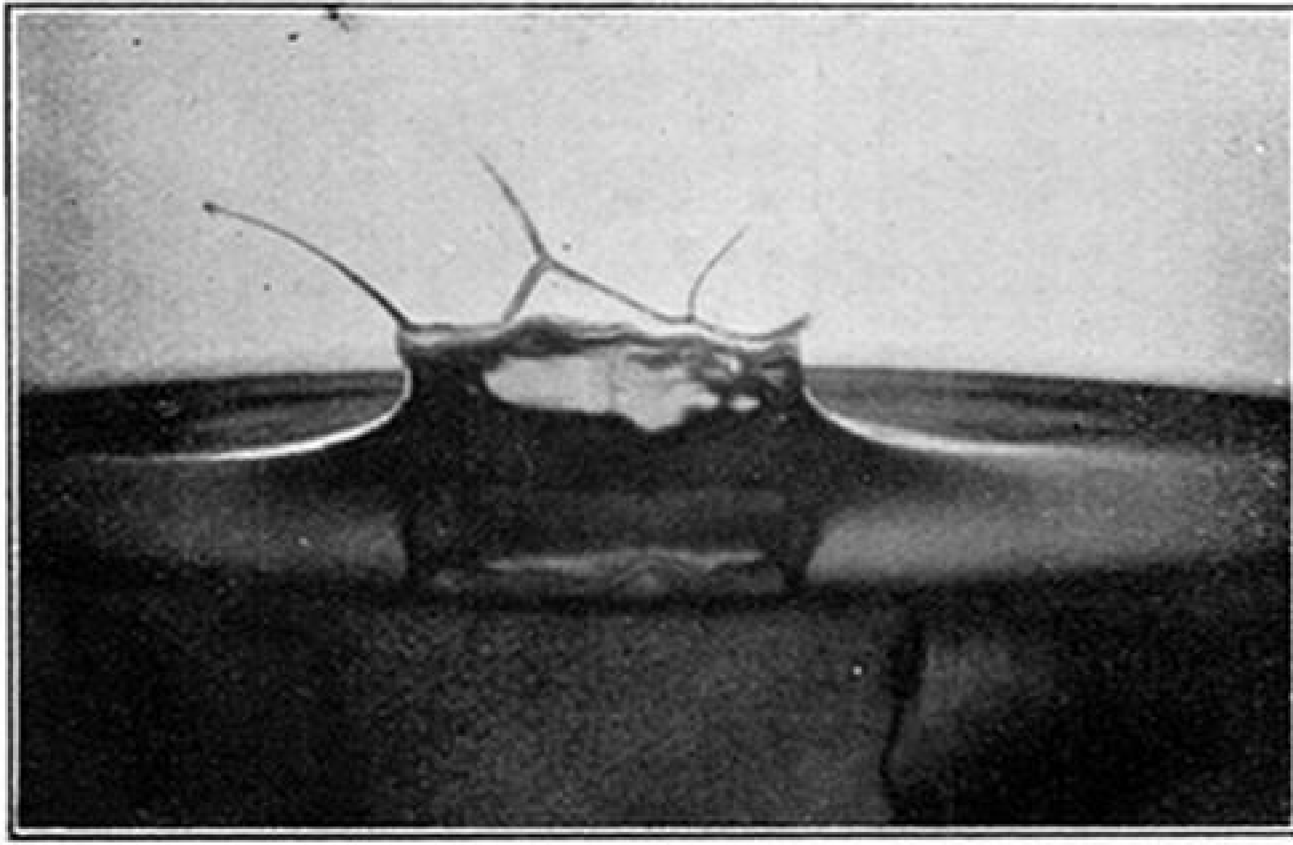


Fig. 2.



Series XXVII.

Fig. 3.



Series XXVIII.

Fig. 1.

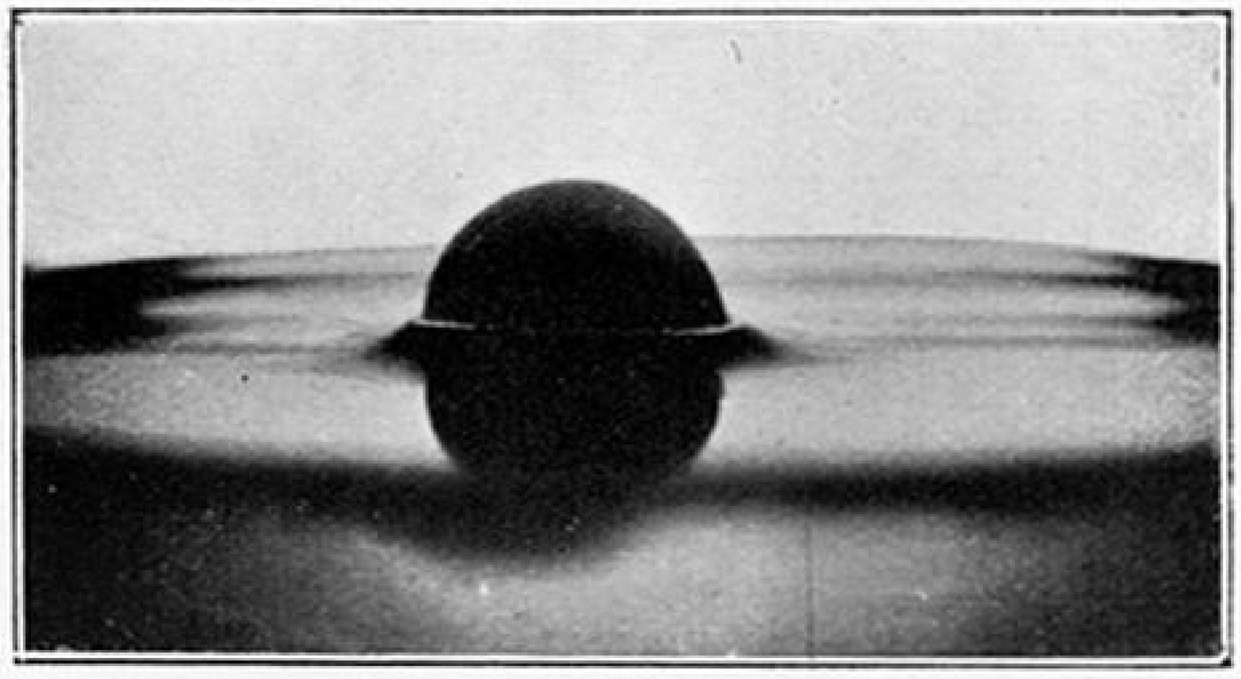


Fig. 2.

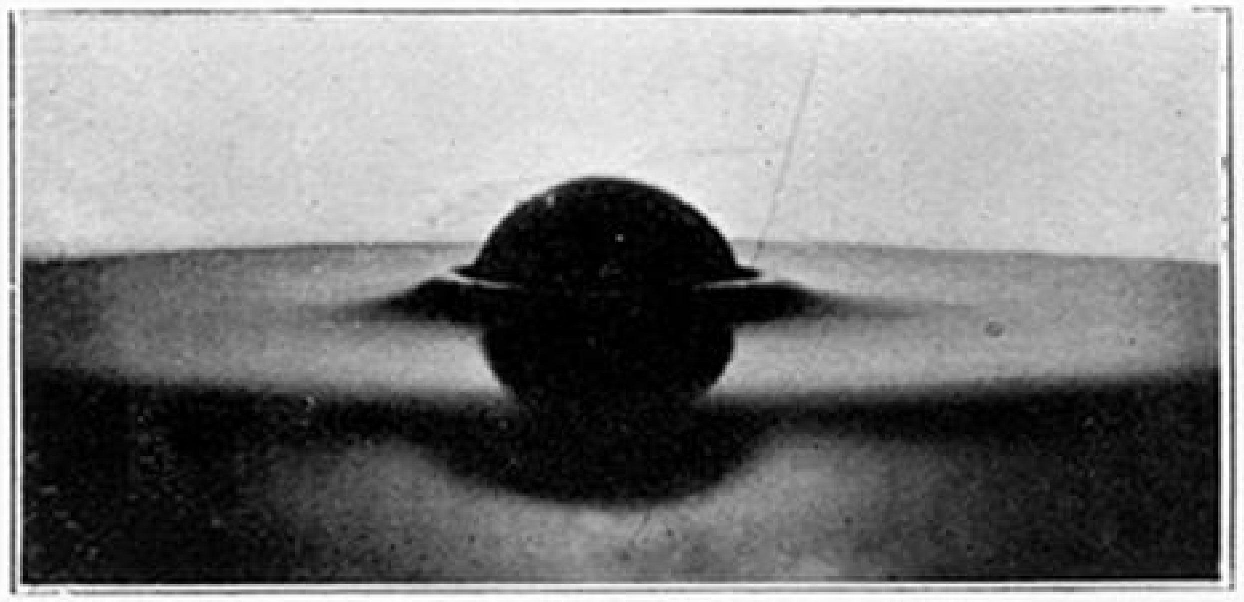


Fig. 3.

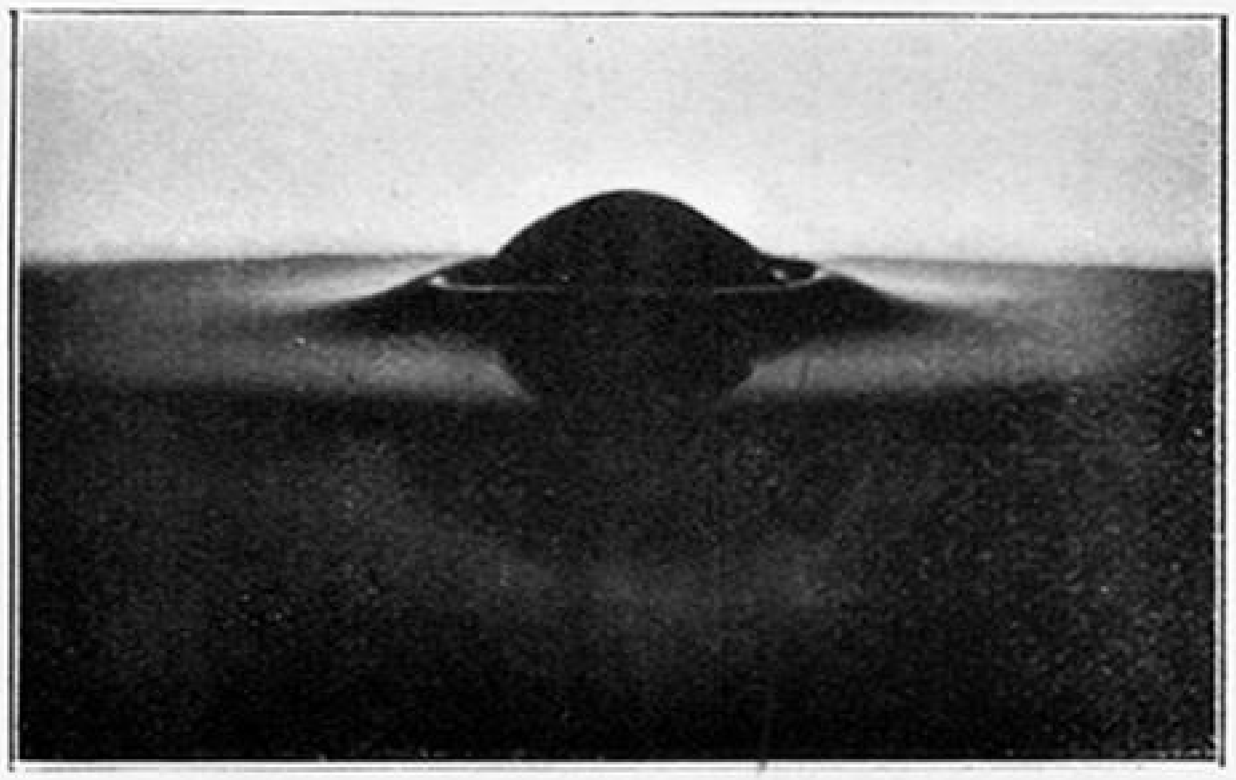


Fig. 4.

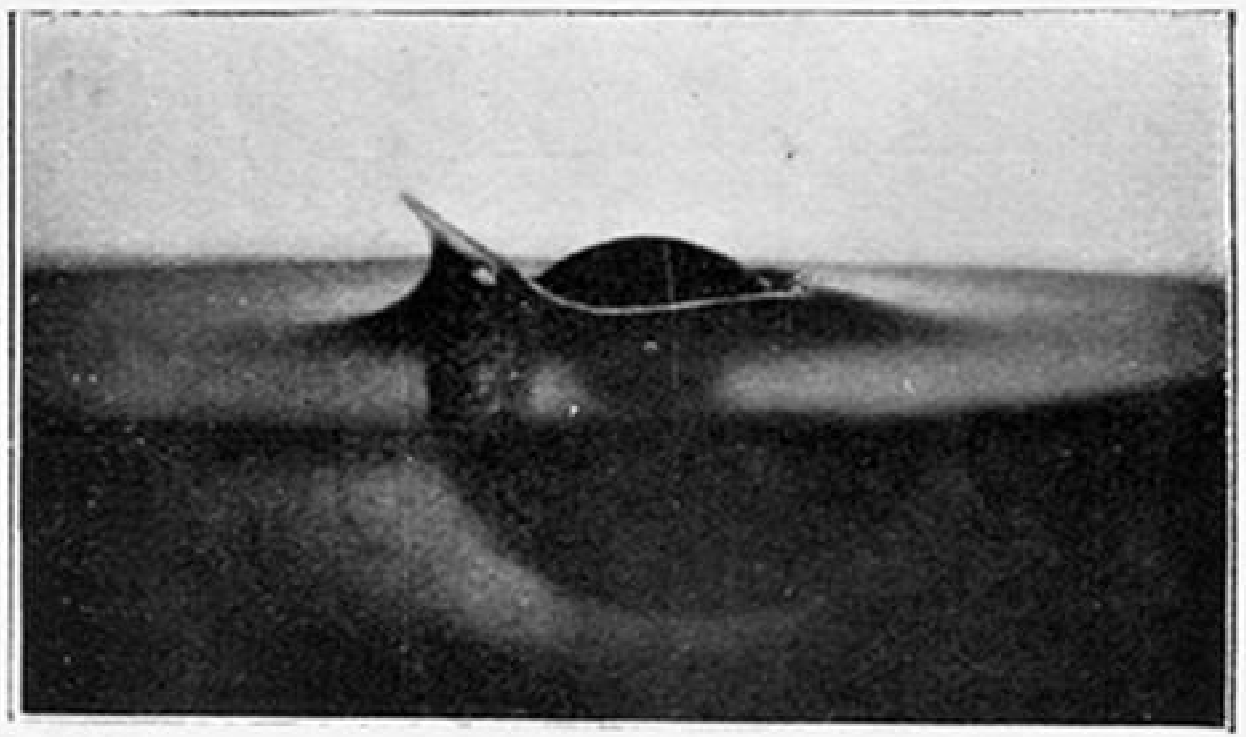


Fig. 5.

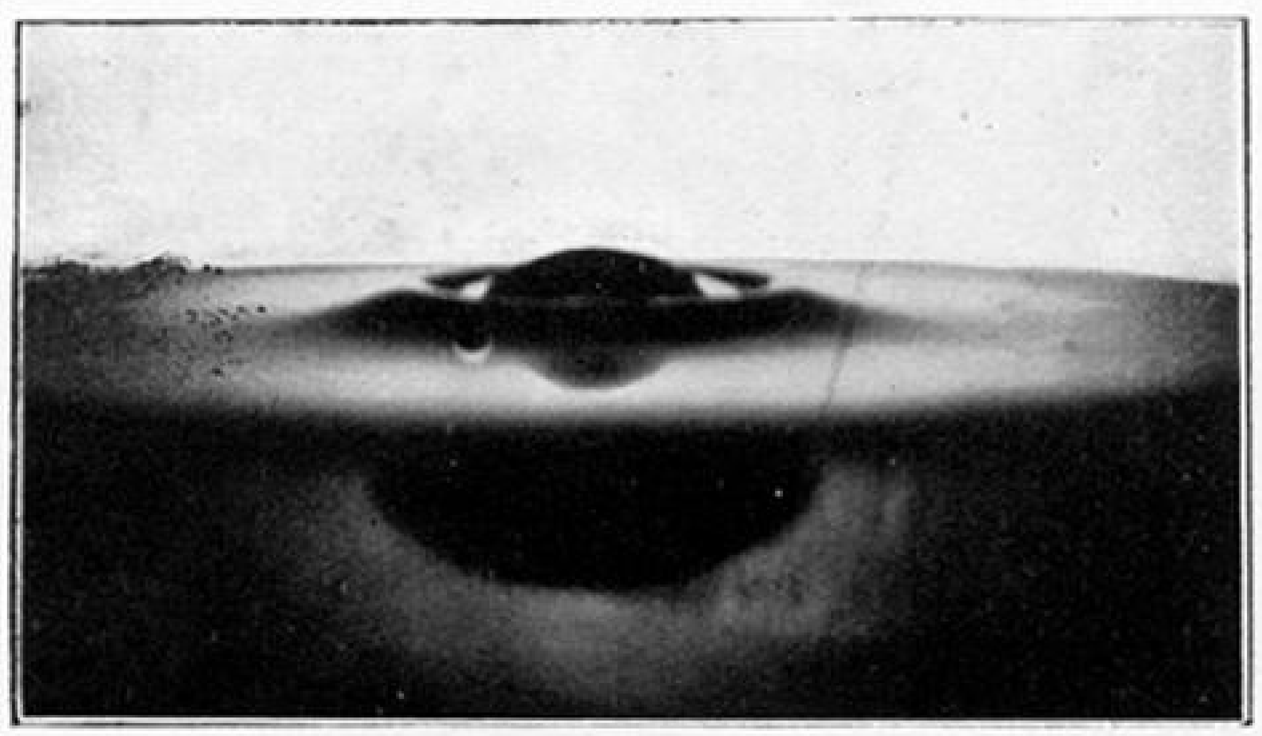
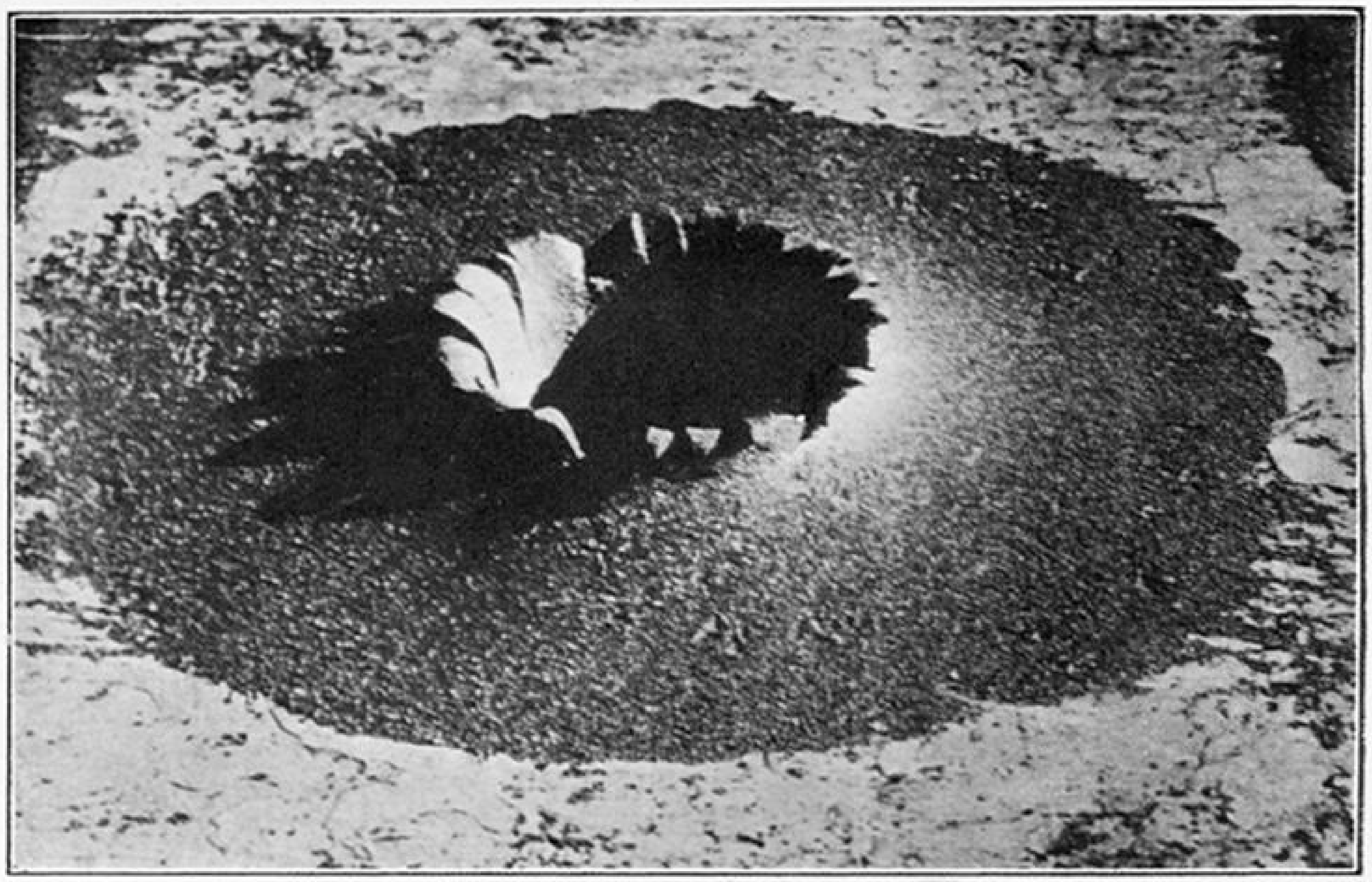
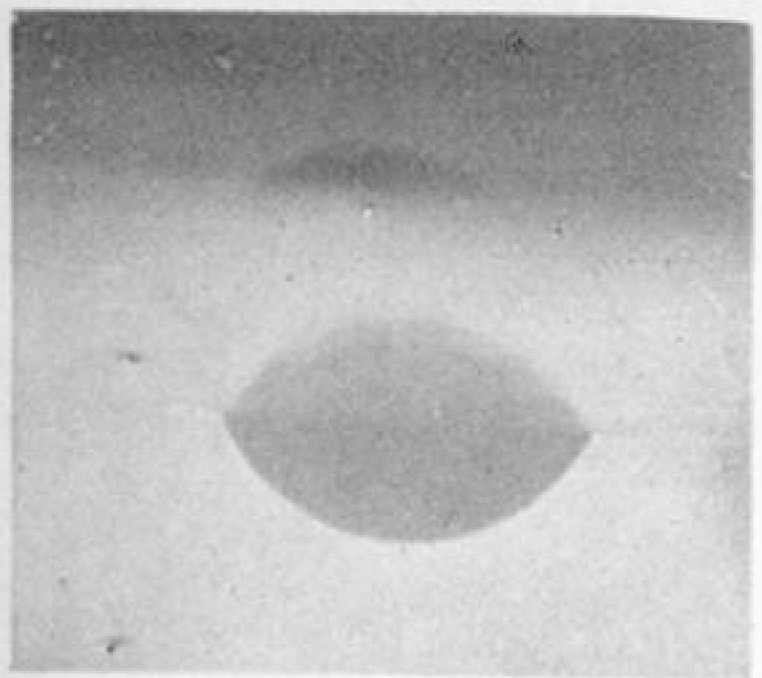


Fig. 5.





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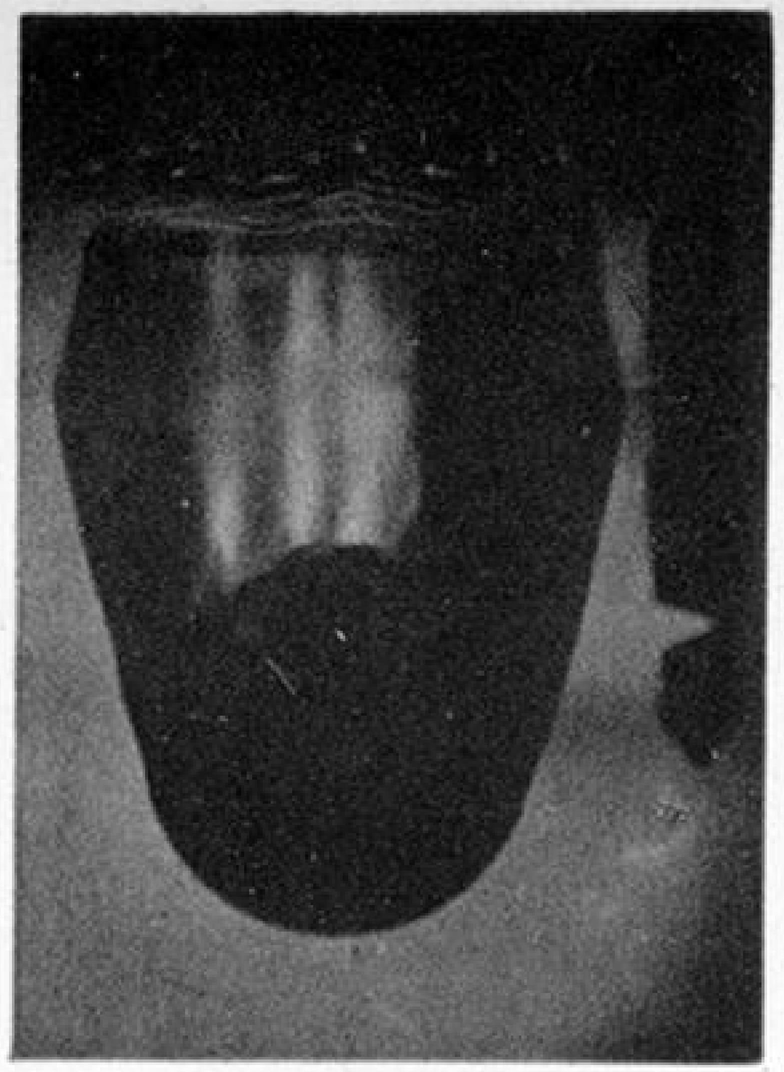

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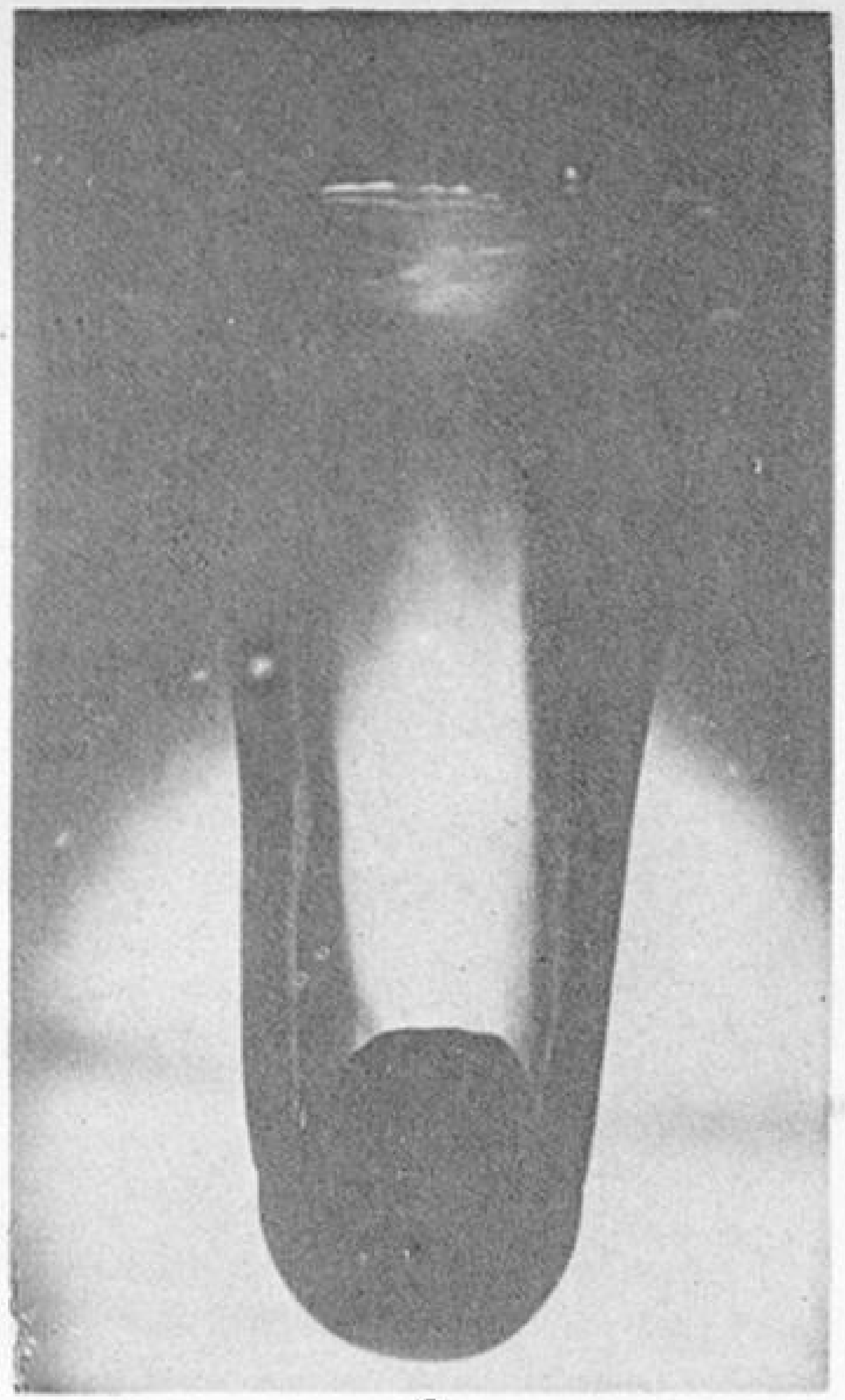

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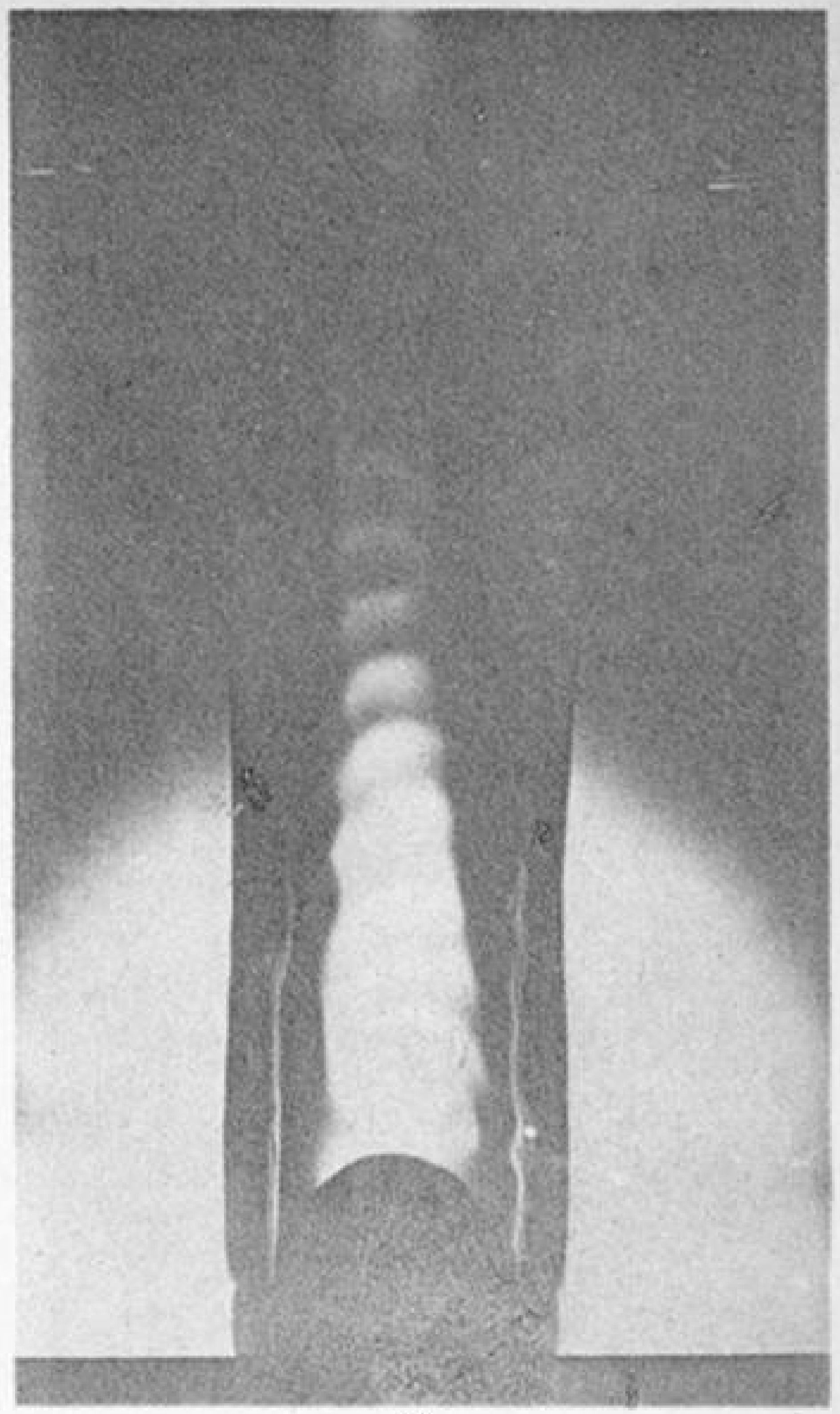

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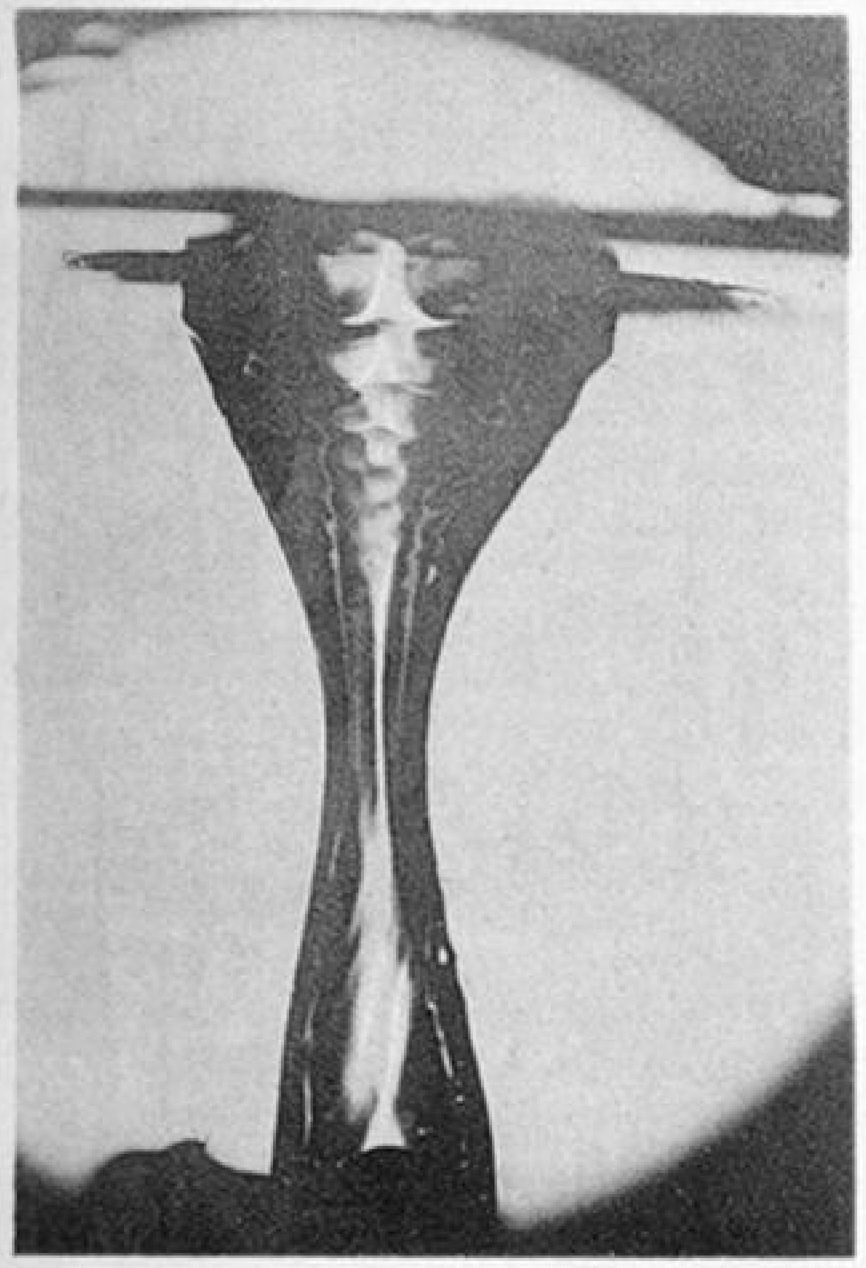

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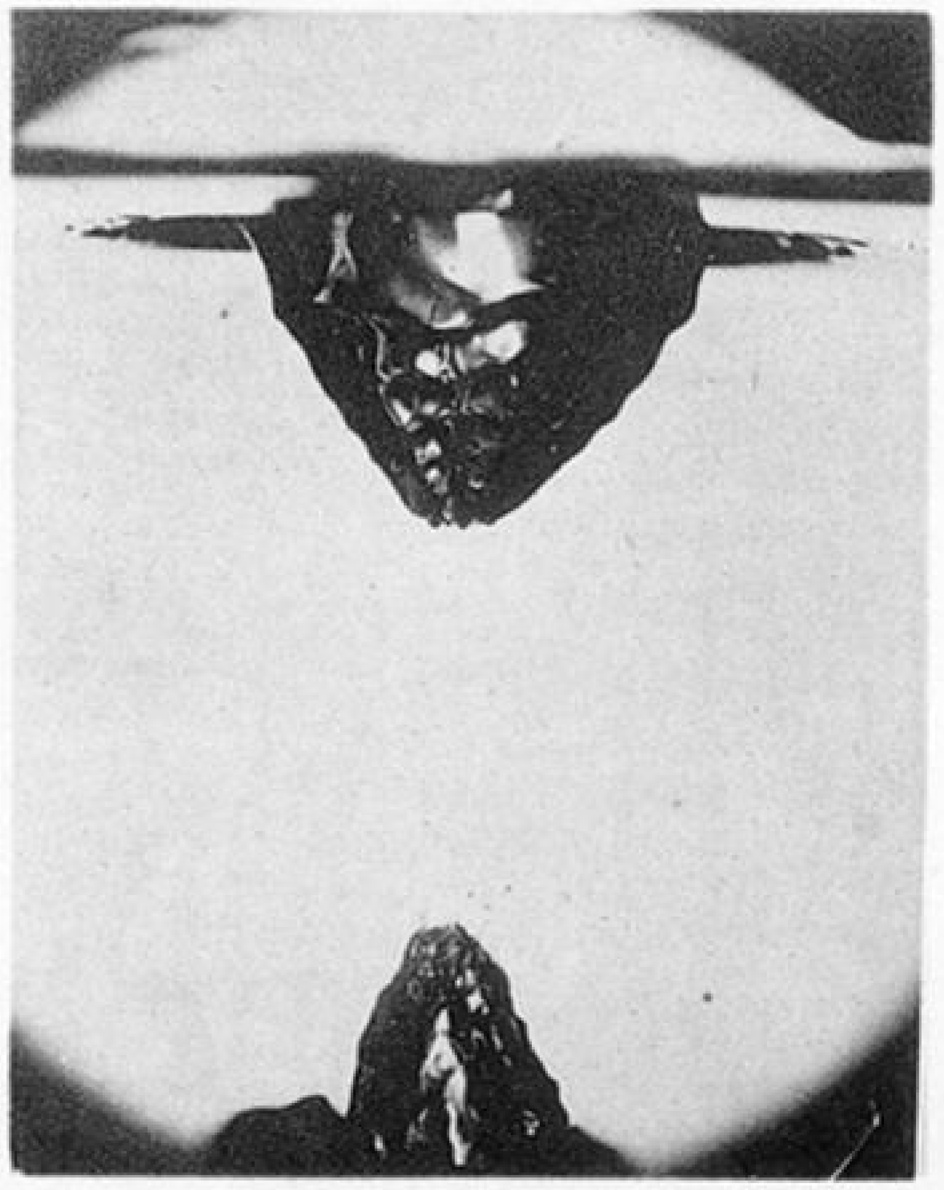

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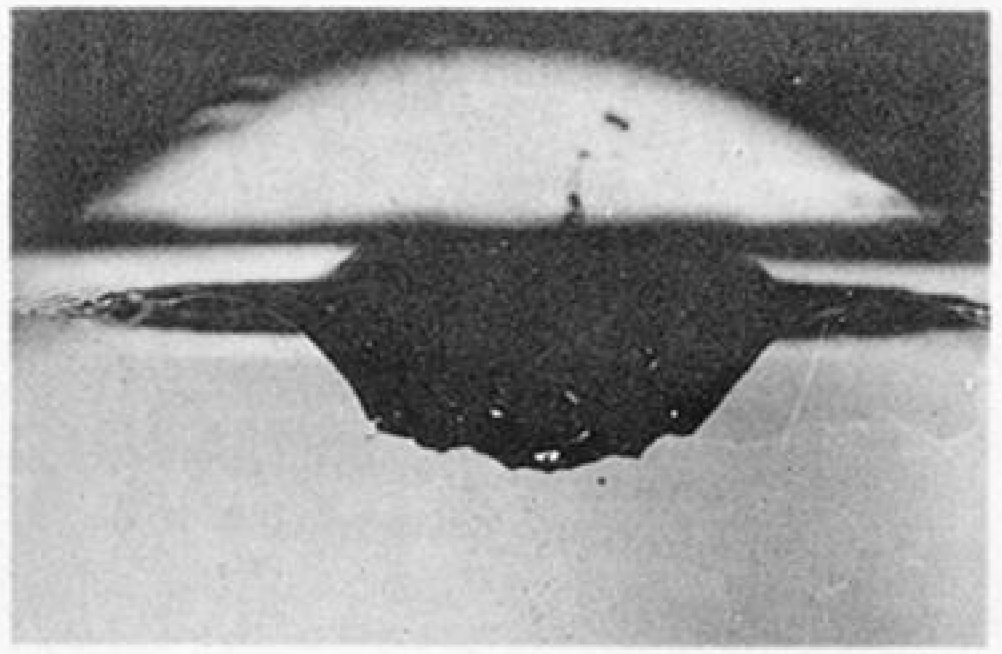

13

16

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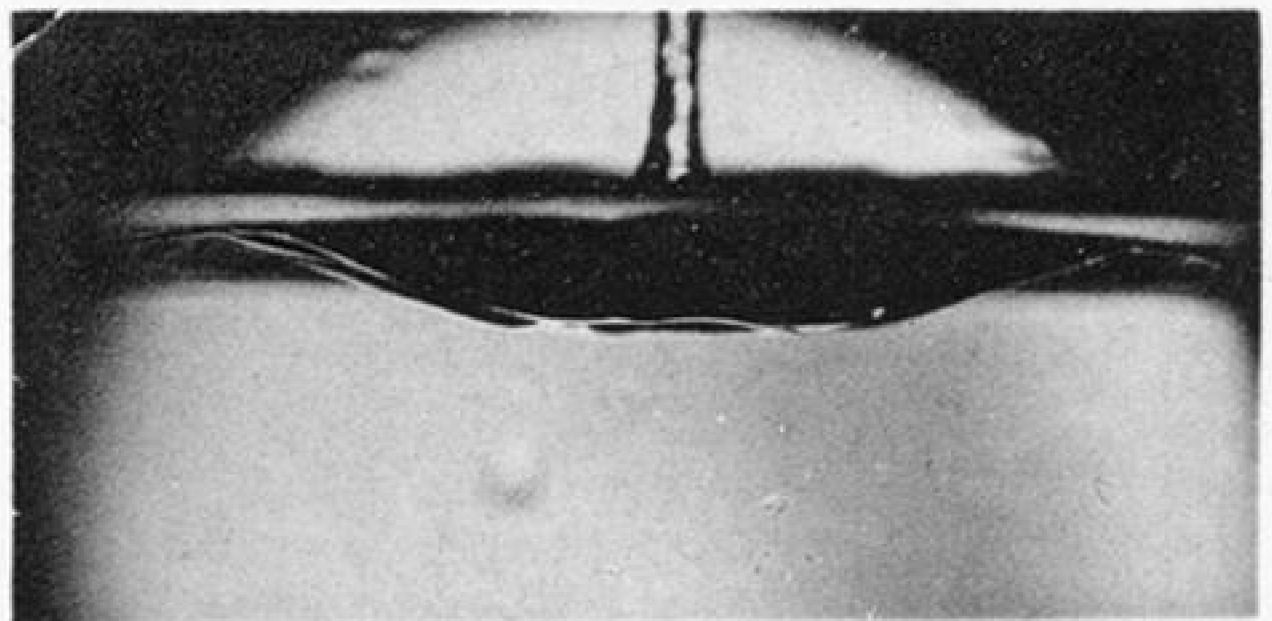
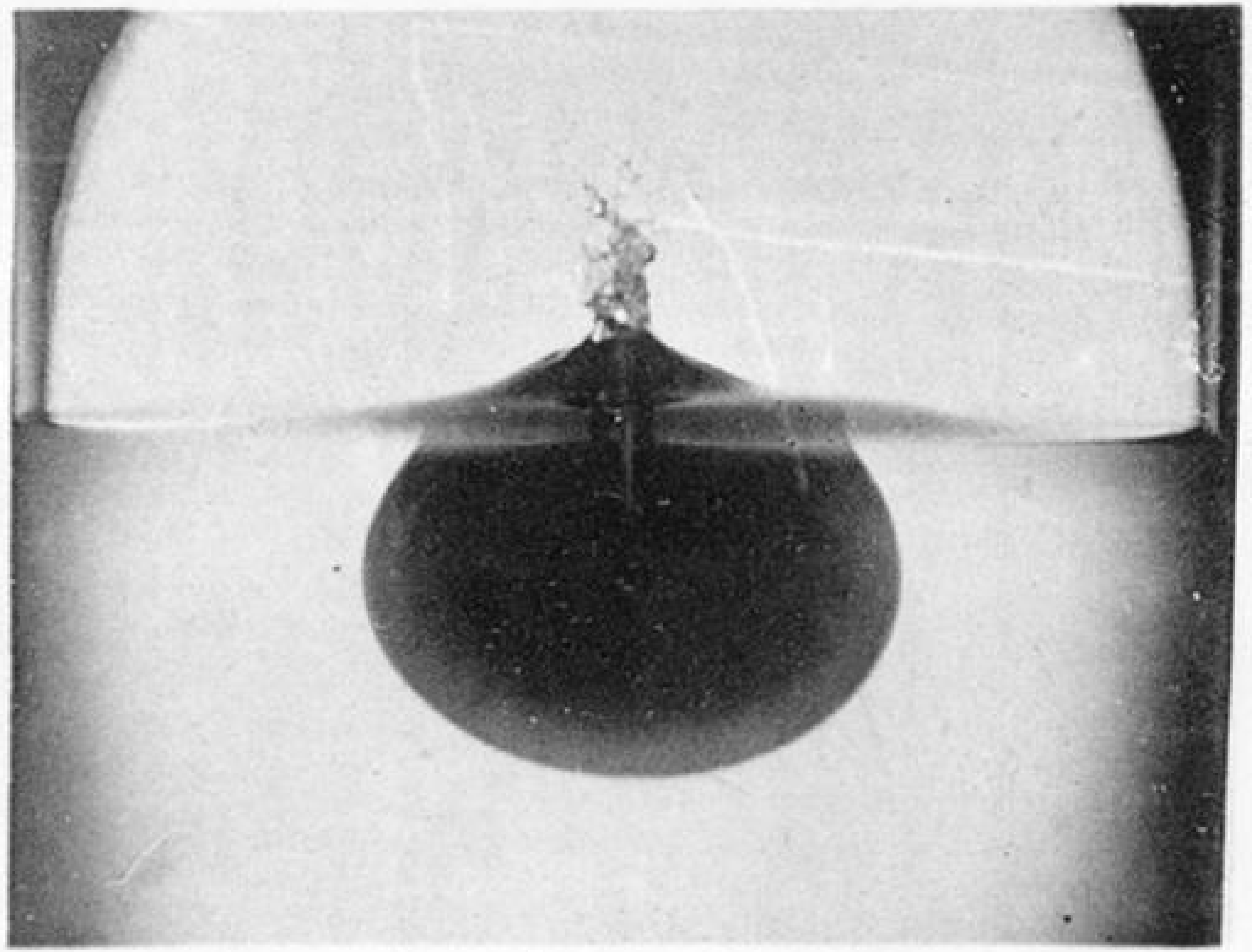


15

17

SERIES XXI.



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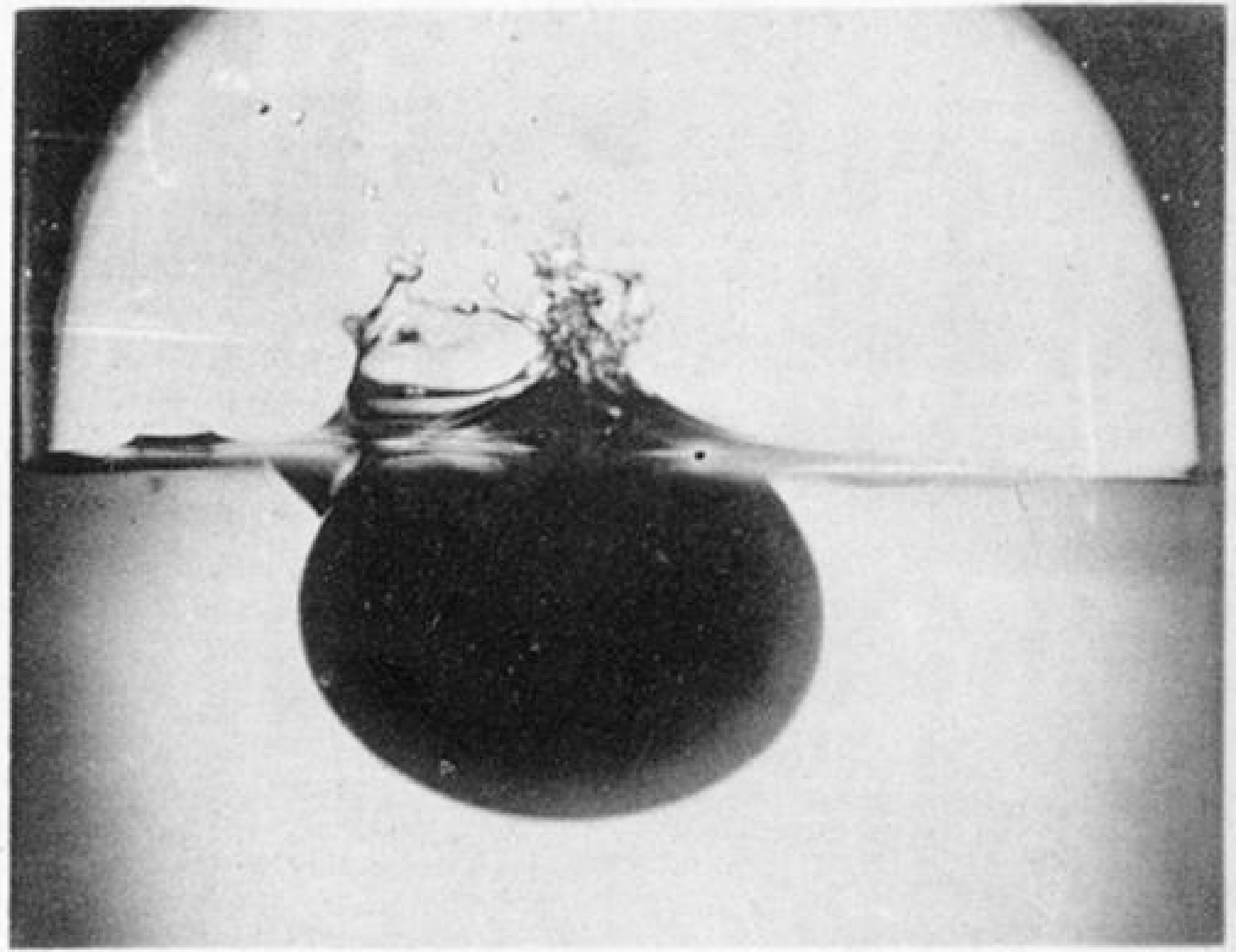

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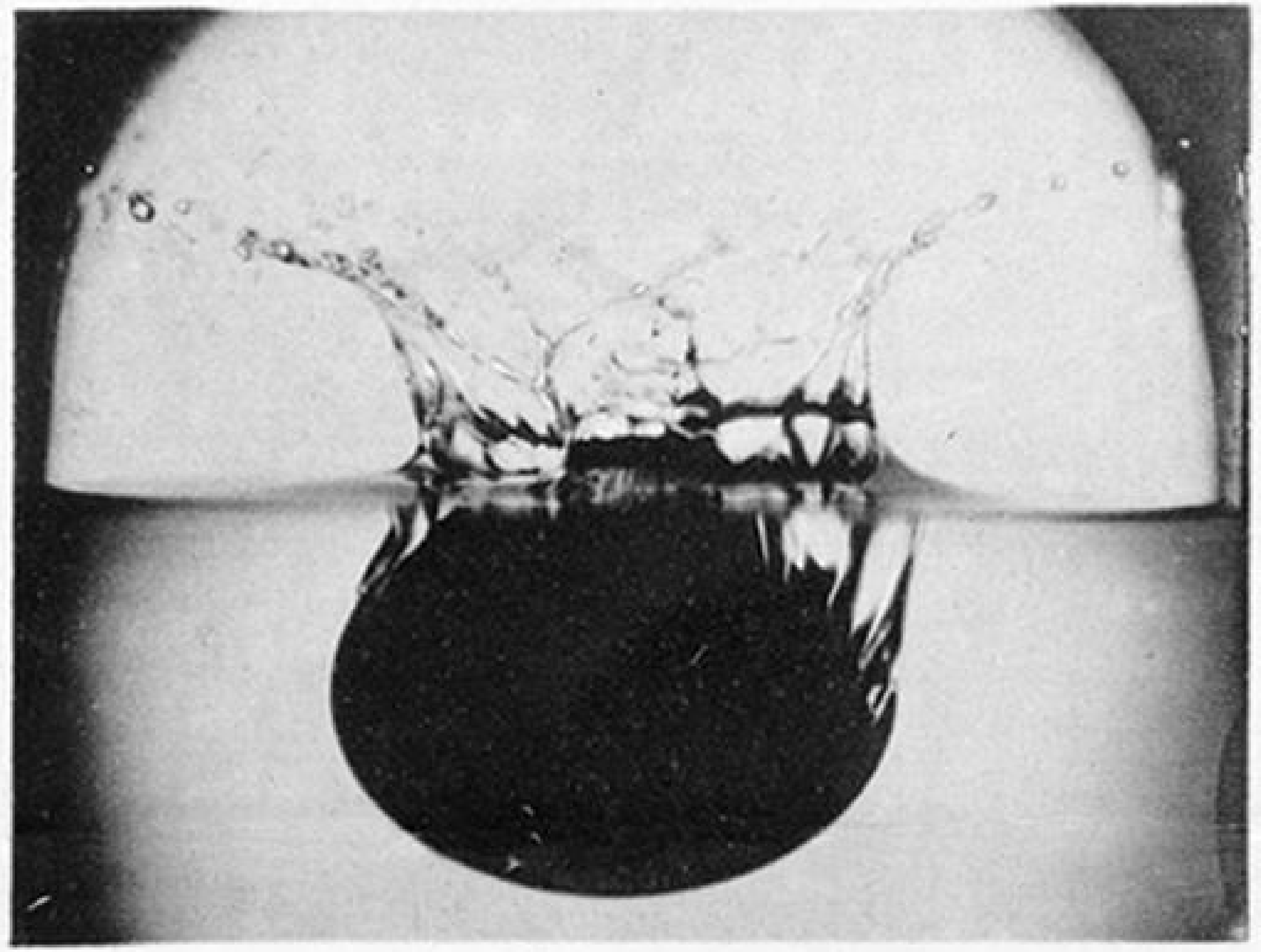

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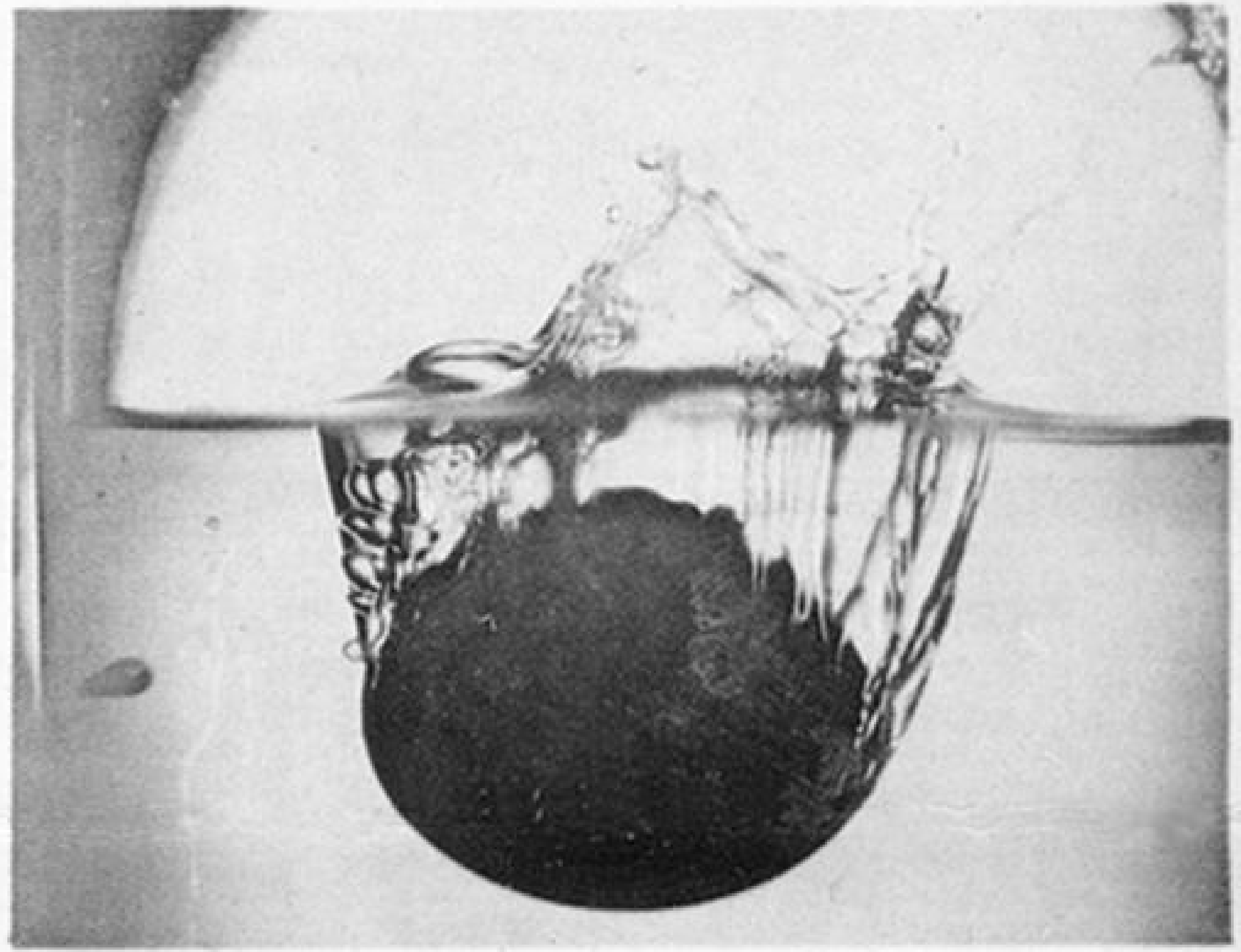
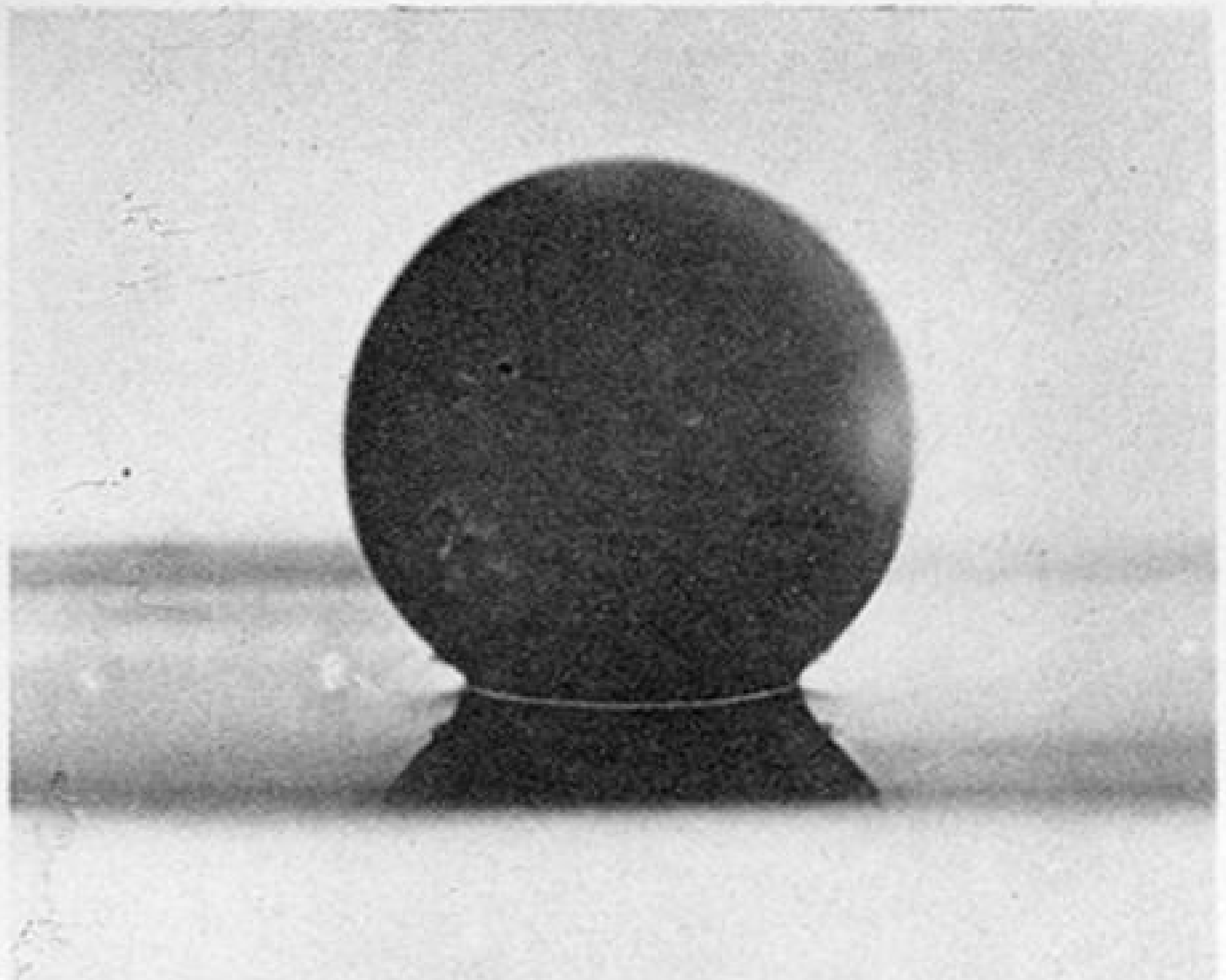


2

SERIES XIII. (15 cm. fall).



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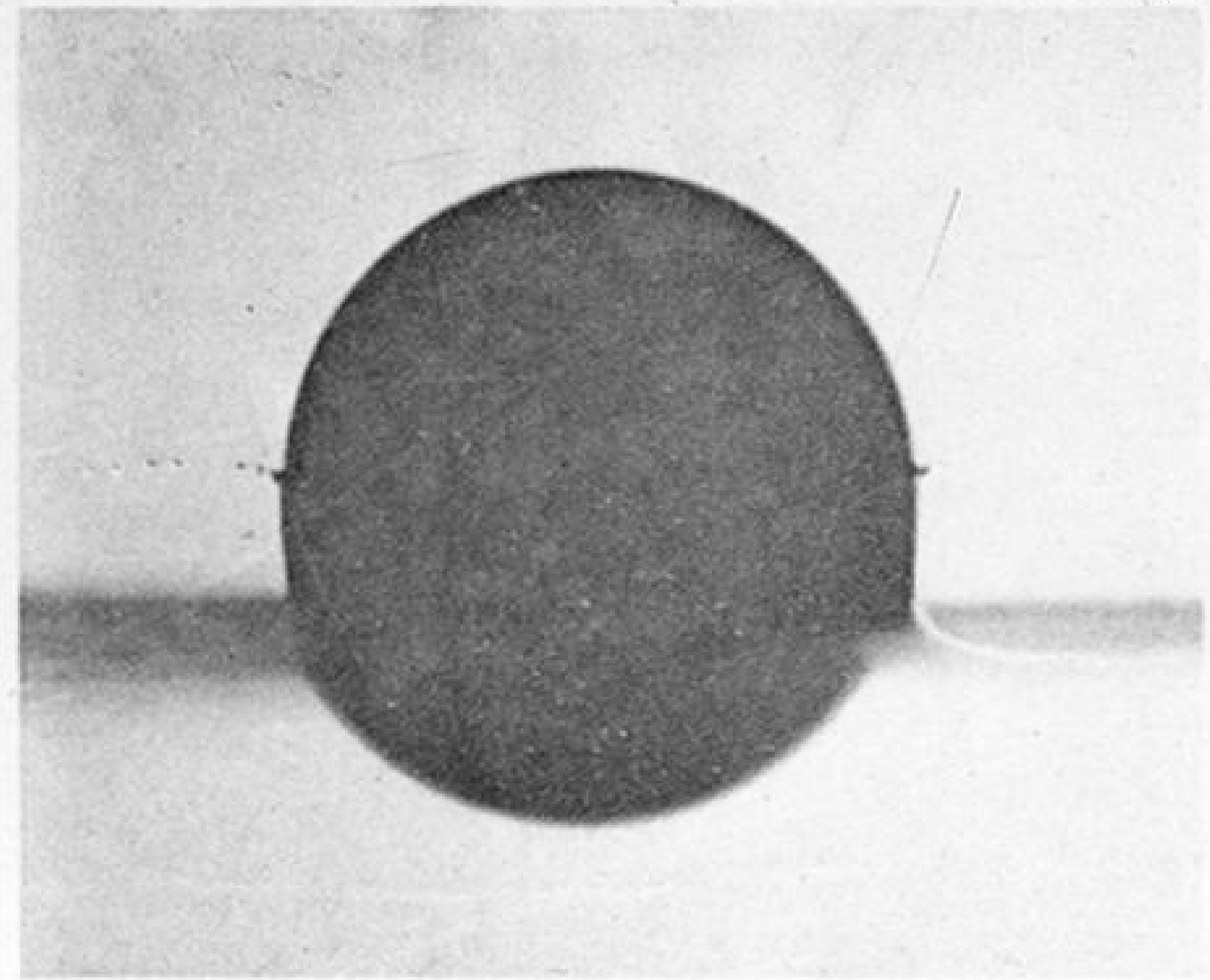

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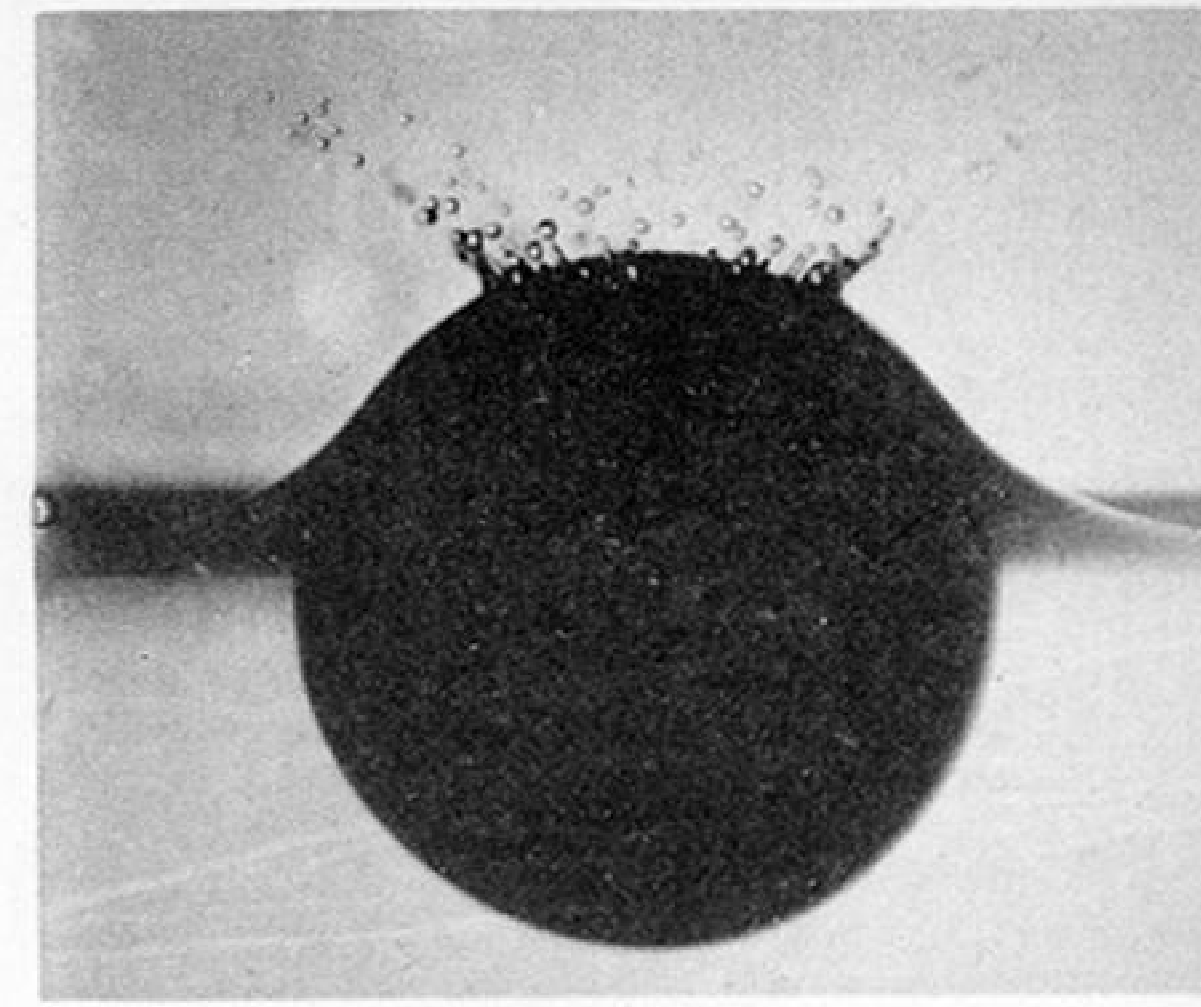

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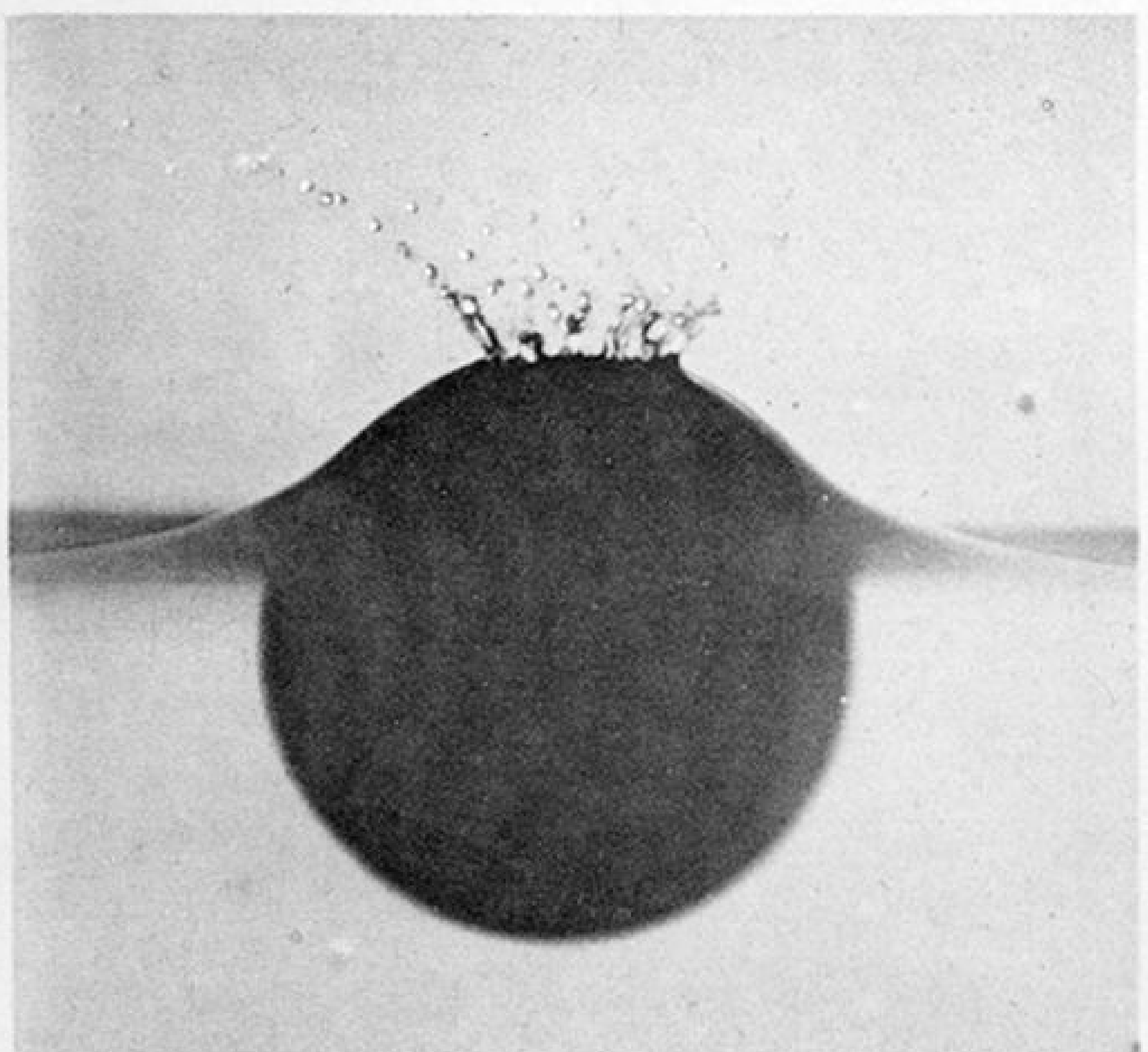

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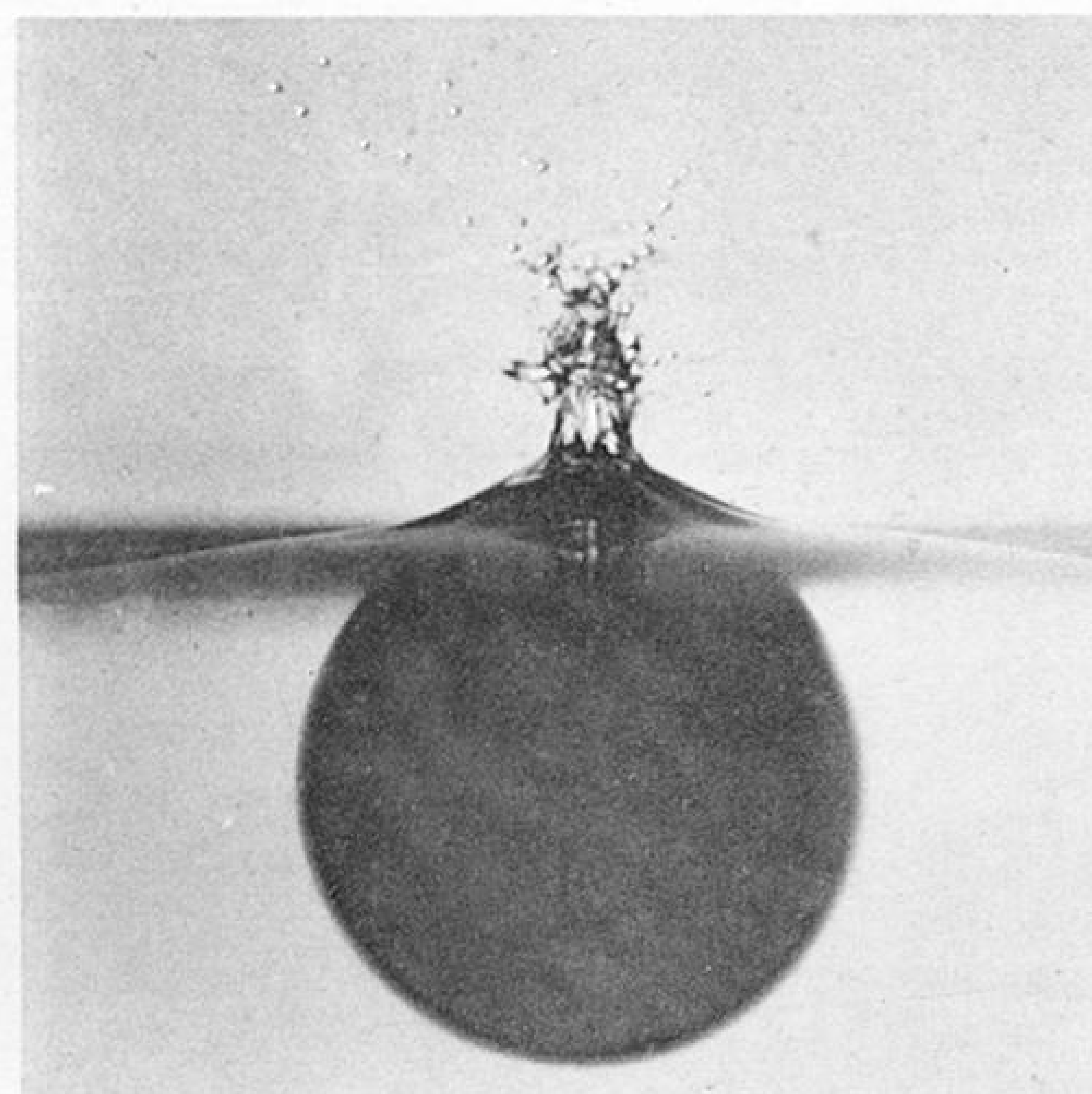
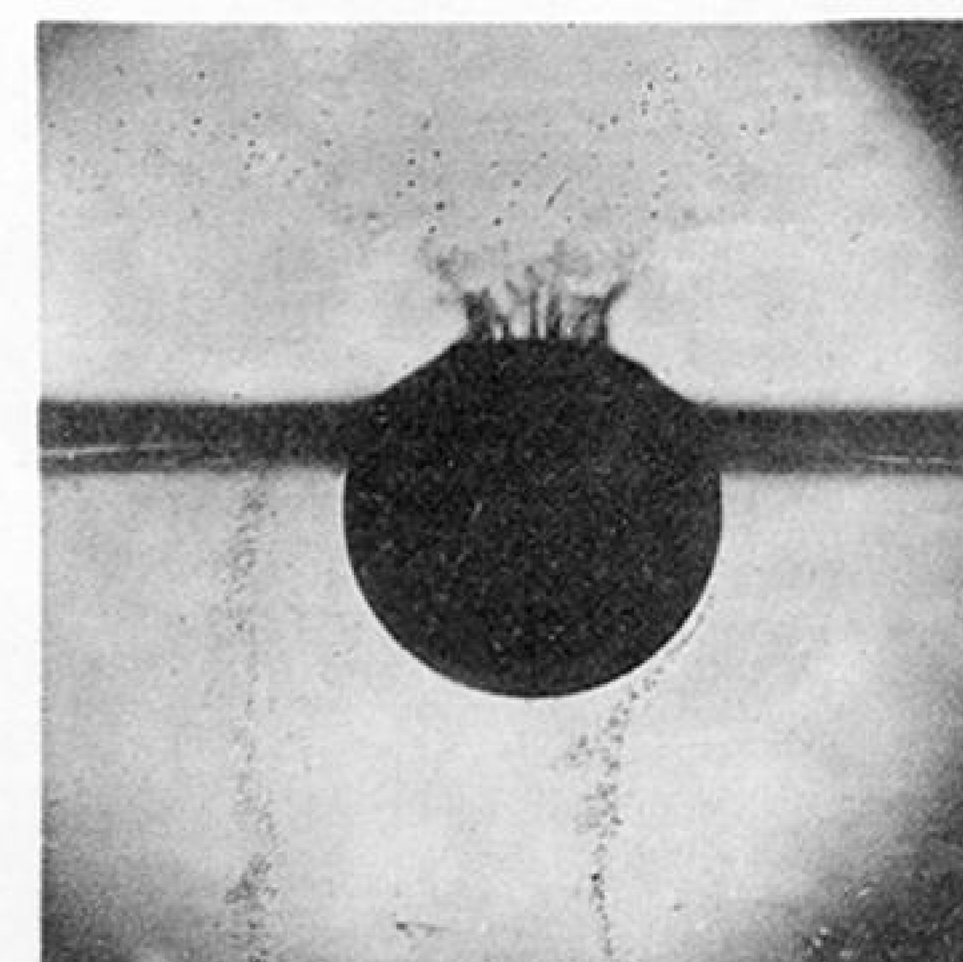
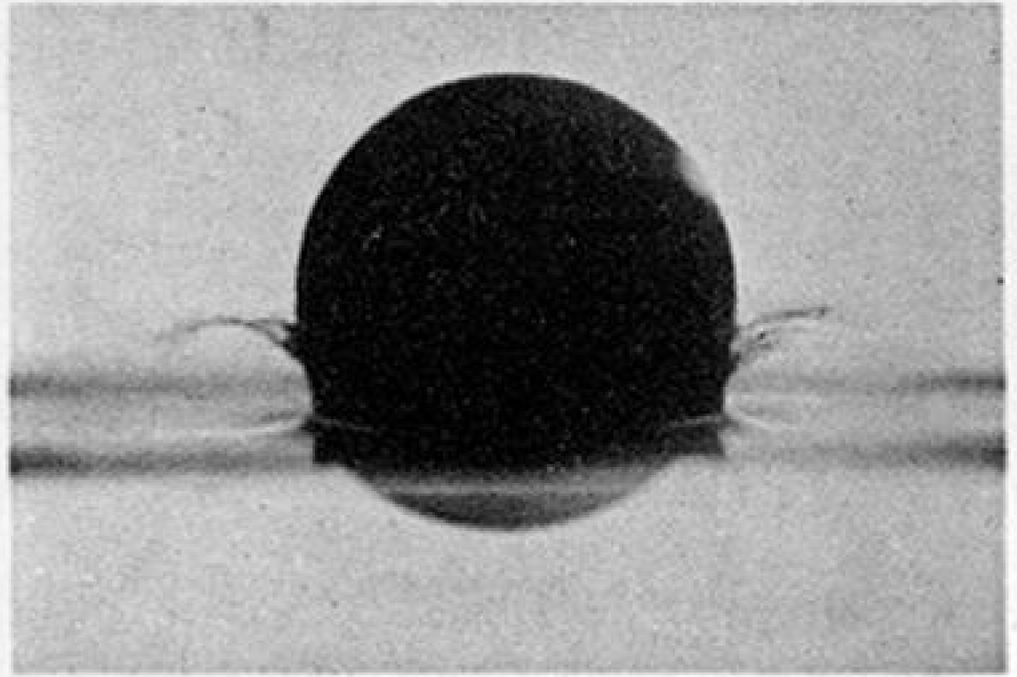


7

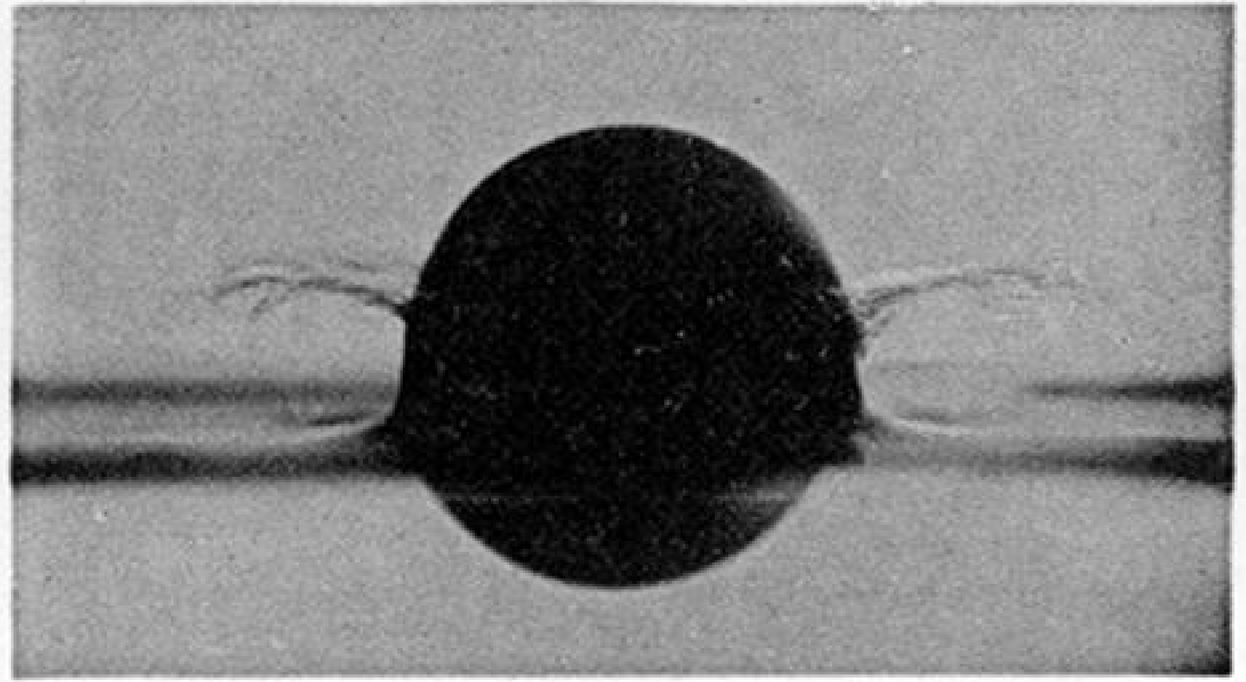


8

SERIES XVI. (100 cm. fall).



1



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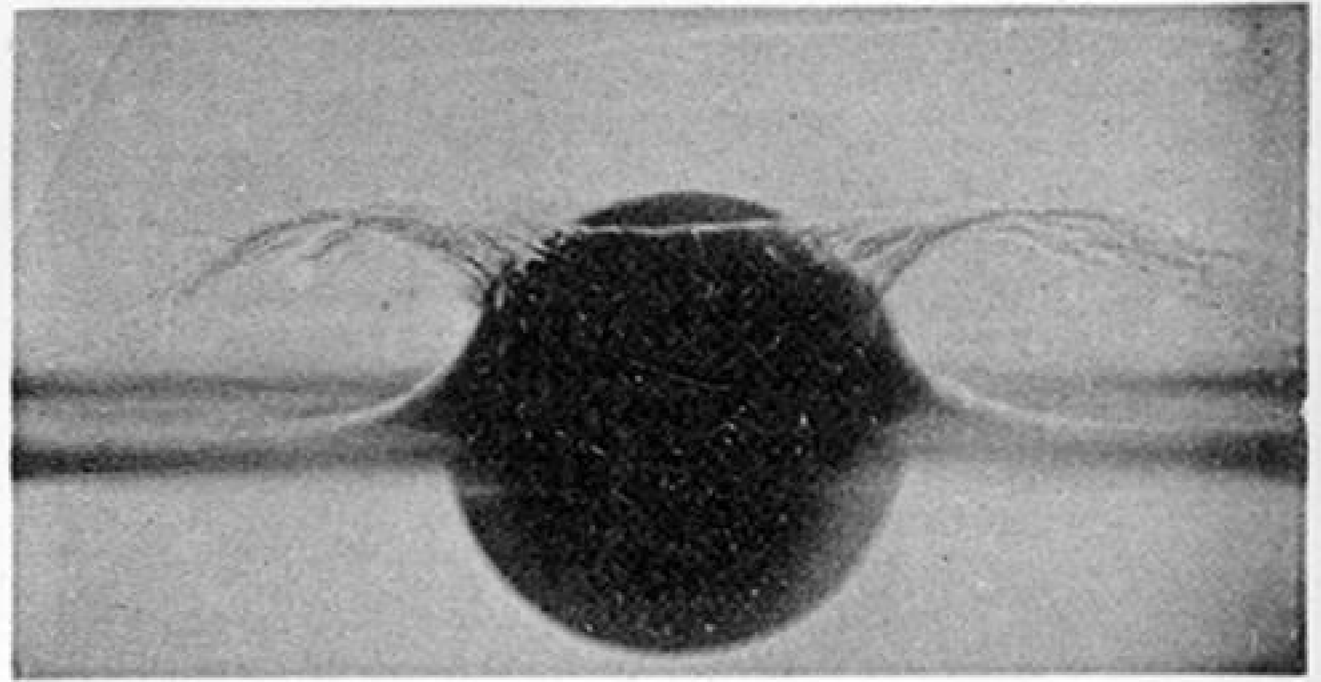

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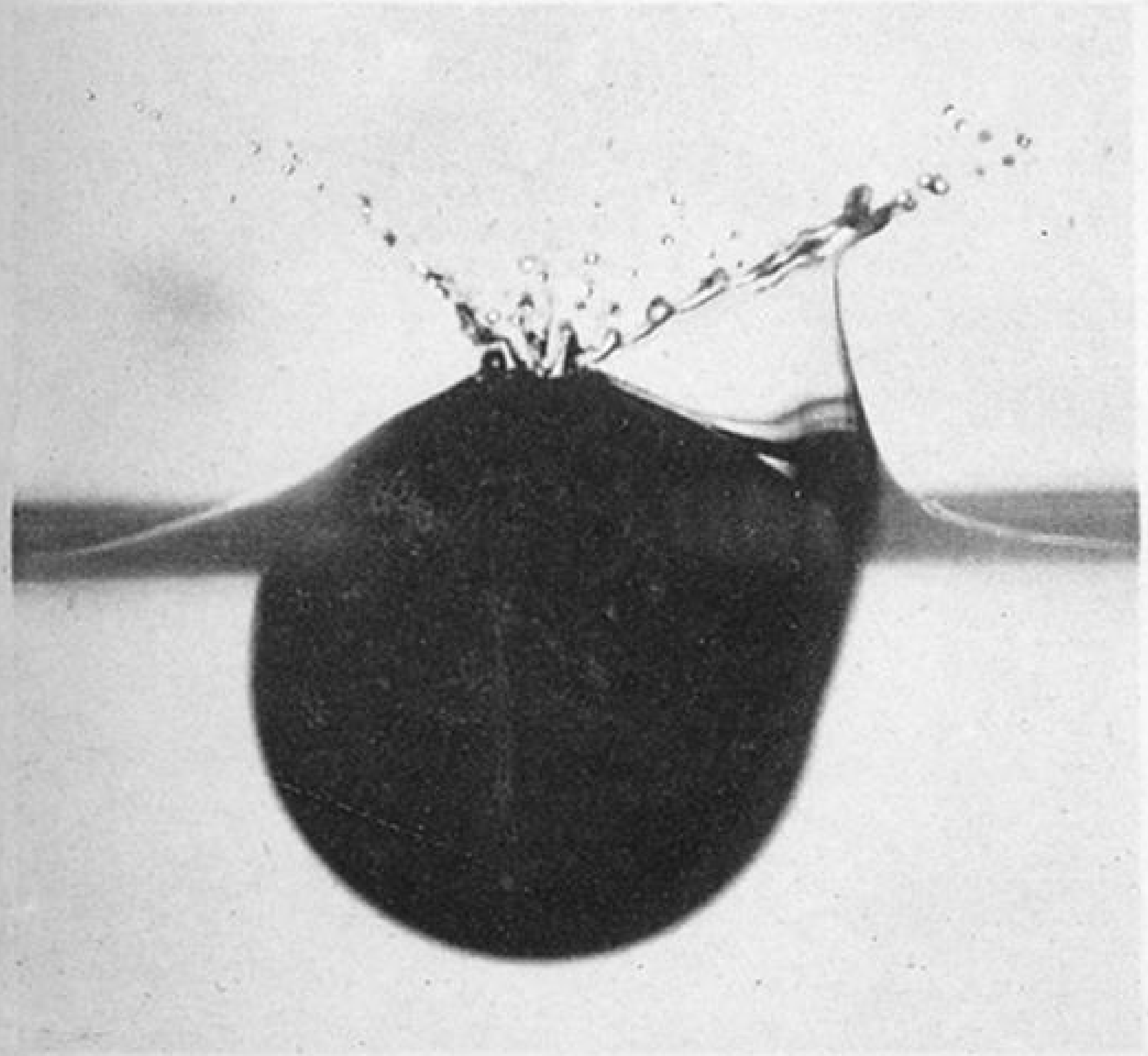
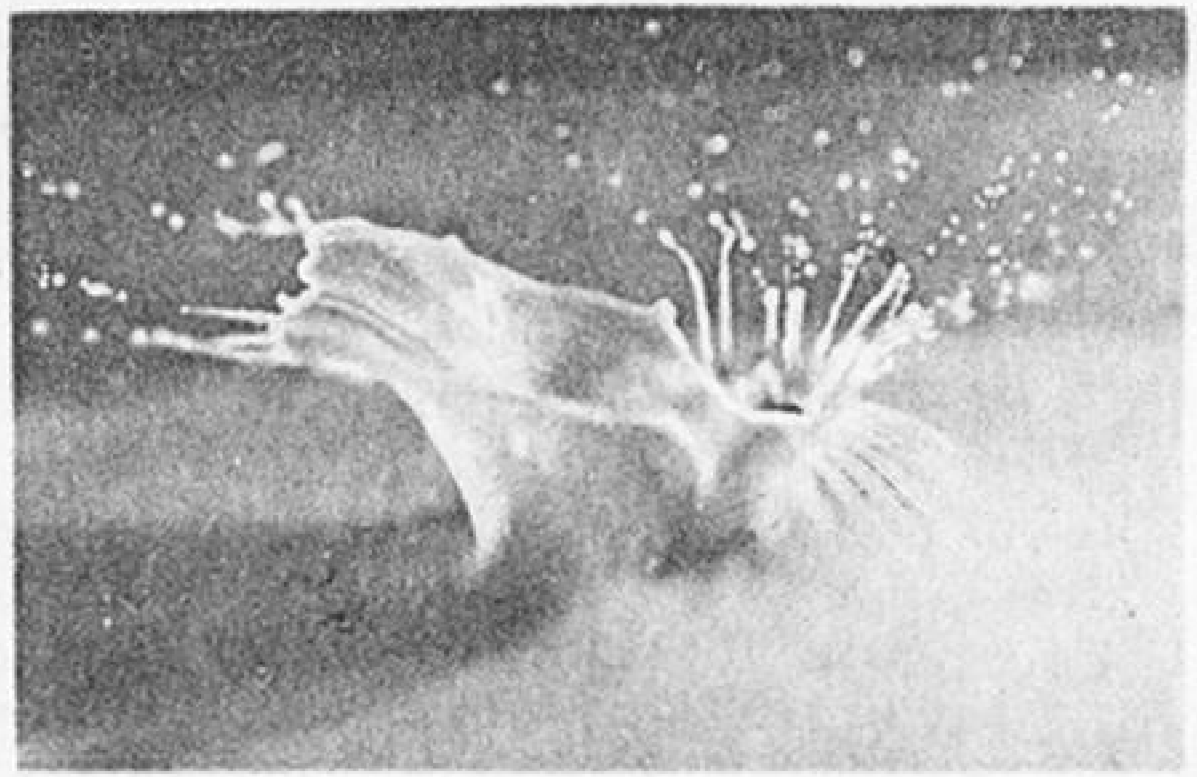
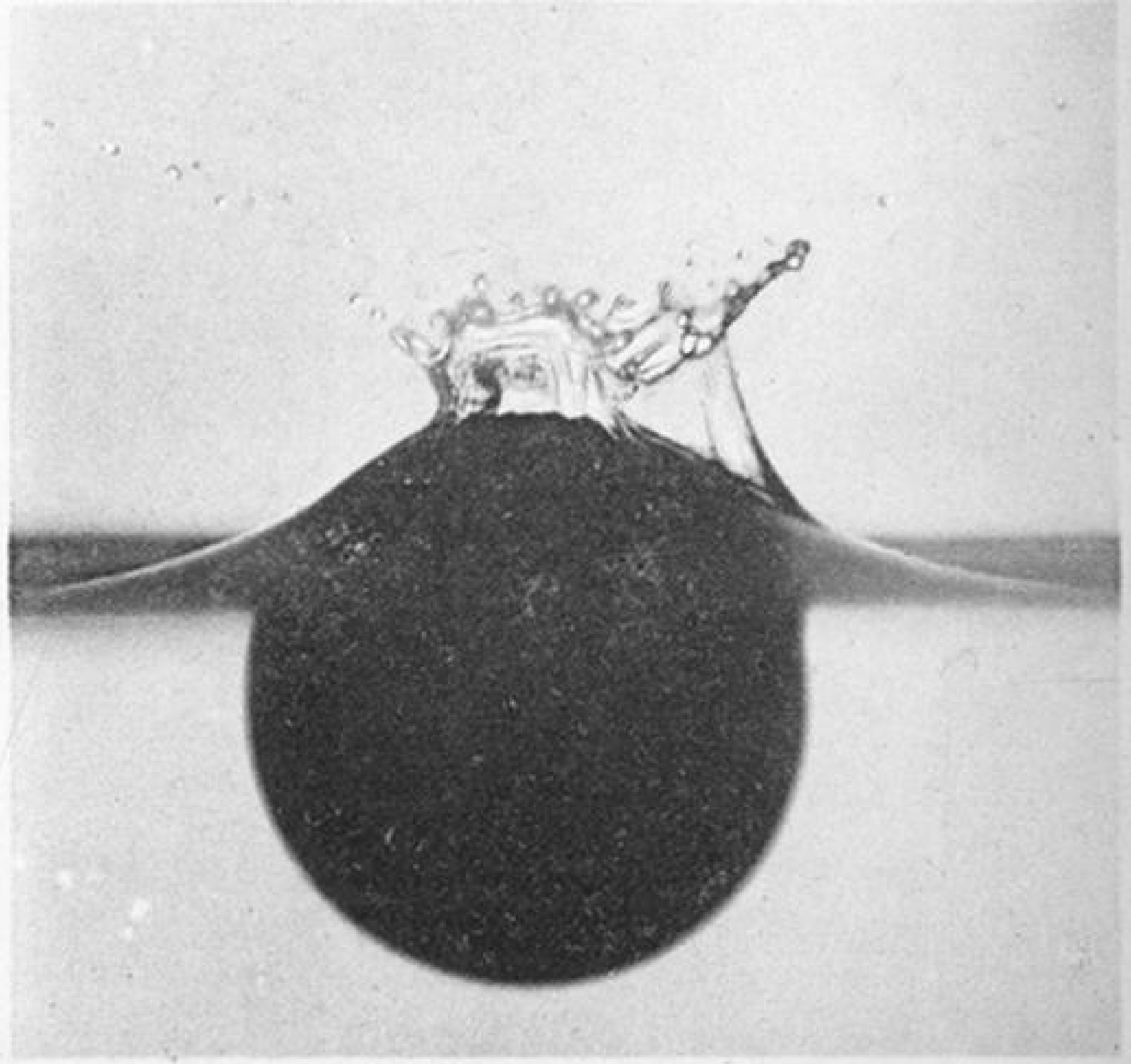


1



2A.

SERIES XX.



2